AN ACTIVE CAMBER CONCEPT FOR EXTREME MANEUVERABILITY: MECHATRONIC SUSPENSION DESIGN, TIRE MODELING, AND PROTOTYPE DEVELOPMENT

A DISSERTATION SUBMITTED TO THE DEPARTMENT OF MECHANICAL ENGINEERING AND THE COMMITTEE ON GRADUATE STUDIES OF STANFORD UNIVERSITY IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF DOCTOR OF PHILOSOPHY

> Shad M. Laws March 2010

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- 1. (Suspension_Design_Software.zip)
- 2. (Prototype_Suspension_Movie.mov)

I certify that I have read this dissertation and that, in my opinion, it is fully adequate in scope and quality as a dissertation for the degree of Doctor of Philosophy.

(J. Christian Gerdes)

Principal Advisor

I certify that I have read this dissertation and that, in my opinion, it is fully adequate in scope and quality as a dissertation for the degree of Doctor of Philosophy.



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(Stephen M. Rock)

Approved for the University Committee on Graduate Studies.

Dedicated to the gearheads of the world: let it be known that you, too, can turn your automotive hobby into a legitimate academic pursuit.

Abstract

Since the beginning of the automobile, engineers have worked continuously to increase its maneuverability. As maneuverable as today's cars have become, they have yet to reach their full potential. The goal of the active camber concept is to take the next step: to generate a vehicle with extreme maneuverability.

The maneuverability of an automobile is limited by tire forces. While tire/road friction does limit the maximum tire forces possible, a significant portion of this friction is not utilized when tires are actuated using steer. However, with camber it is possible to generate up to 30% more lateral tire force. Therefore, active control of camber, in coordination with active steer and suspension, is used to maximize lateral force capability. The result is a more maneuverable vehicle with increased turning capacity.

The contributions of this dissertation follow two main threads. The first is active camber tires. Passenger car tires are not well-suited for use at high camber angles, and motorcycle tires do not exhibit large gains in peak lateral force by using camber as opposed to steer. To exploit fully the benefits of cambering, the active camber concept requires new, specialized tires. To accomplish this, a new, 2D variant of a brush tire model is developed that considers the force distribution in the contact patch in both lateral and longitudinal directions. This gives a more complete picture of how camber forces are generated than existing models, which typically ignore the lateral distribution. Similar to other brush models, this requires a model of the vertical force distribution in the contact patch. Several contact patches are measured and characterized using a new, semi-empirical contact patch model. The resulting 2D brush tire model is used to characterize existing motorcycle tires and to predict what design parameters need to be altered to generate a tire that *does* exploit fully the benefits of cambering.

The second thread of this dissertation is active camber suspension systems. Of course, a conventional suspension system will not suffice for the active camber concept: a simple mechanism connected to a steering wheel will not be sufficient to coordinate the steer and camber angles of all four wheels to maximize lateral force. A specialized, mechatronic suspension system is required that provides full control over the tire to maximize maneuverability. However, the design *criteria* presented by existing suspension design literature do not address active camber. Therefore, this dissertation articulates a clear set of design *principles* rooted in the design of mechatronic systems and applied to a kinematic model of the suspension system. By using the forward kinematics, inverse kinematics, and Jacobians of the model, the design principles are mapped into design criteria. By applying this process to conventional suspension systems, design criteria are developed that are similar to existing suspension design literature. By applying this process to a suspension system with active camber, active steer, and active vertical suspension, design criteria for the active camber concept are developed. These are then used to guide the design and construction of a prototype suspension system.

The prototype suspension system is attached to rollers on a chassis dynamometer, providing an experimental rolling road for testing. This is used to measure the performance of three different motorcycle tires. Not only does this data serve to validate the new tire model, but it also serves to demonstrate the capability of the suspension system as a research testbed. This can be used to further the development of an active camber concept, providing vehicles with more lateral force capability than anything else on the road. The result: a vehicle with extreme maneuverability.

Acknowledgments

While I am listed as the sole author of this dissertation, it would be somewhere between a gross exaggeration and an outright lie to say I did it alone. There were many challenges along the way, and it was thanks to my wonderful circle of friends, family, and colleagues that I was able to complete each of them. I owe a great deal of thanks to a great number of people.

One such challenge was the fabrication of the active camber prototype, which is central to the results of my thesis. To realize my design, I was lucky enough to enlist the help of many great fabricators, including Eric Gauthier of Suspension Performance in Palo Alto, Chris Wakim of Race Concepts in Simi Valley, and John Buddenbaum of Buddenbaum Fabrication in San Carlos. I was also lucky enough to have the help of several people at Stanford, including Preston Clover, Peter Kardassakis, Alex Kwan, Jackie Liao, Hyungchai Park, and Scott Sutton. Thanks to all of you for helping make what is certainly one of the world's most sophisticated and wild suspension systems!

My Ph.D. had an additional big fabrication challenge: P1, our beloved, all-electric, by-wire vehicle testbed. While it makes no appearance in this dissertation outside of Figure 1.1, this project dominated the first half of my Ph.D. and provided me with invaluable experiences that, among other things, helped influence and shape the active camber concept. This experience would not have been possible without the support of both my lab, the Dynamic Design Lab (DDL), and the Product Realization Lab (PRL). Specifically, I'd like to thank Dave Baggeroer, Craig Beal, Carrie Bobier, Will Crump, Quin Garcia, Scott Kohn, Craig Milroy, Allegra Shum, Jon Thomas, and Paul Yih for their help with P1. I'd especially like to thank Chris Gadda, who along with myself was the Ph.D. student chiefly responsible for its construction. Together, we all made what is one of our fields' most prolific experimental testbeds - thank you so much!

I had significant help behind the scenes of these projects as well. When I encountered (or created) administrative hurdles, it was Denise Curti and Jennifer Rahn who helped me out - thank you so much. Additionally, I'd like to thank Janet Beegle, and L. S. "Skip" Fletcher, T. J. Forsyth, and Stephen Patterson at NASA Moffett Field for making it possible to use their airfield as our proving grounds. Also, I'd like to give a sincere thinks to Nissan Motor Corporation for being an amazing sponsor and supporter of *both* of my Ph.D. projects. Specifically, I'd like to thank Toshimi Abo and Kazutaka Adachi for their support of the P1 project and Yoshitaka Deguchi and Shinichiroh Joe for their support of the active camber project. It has been my pleasure to work with you over the past several years, and I can only hope that my input to your work has been as valuable as your input to mine.

I also owe a big thanks to my lab group, Dynamic Design Lab. In addition to lab members mentioned above, I'd also like to thank Hong Bae, Holly Butterfield, Sam Chang, Rami Hindiyeh, David Hoffert, Adam Jungkunz, Krisada "Mick" Kritayakirana, Hsein-Hsin Liao, Nikhil Ravi, Matt Roelle, Jihan Ryu, Matt Schwall, Greg Shaver, Josh Switkes, and Kirstin Talvala for helping make DDL one of the best labs around. To me, the single moment that best epitomizes how I feel about this group is seen in Figure 1: on the Monterey Coast where we ran the IFAC Symposium on Advances in Automotive Control, cheering to its great success. Because of neverending helping hands, entertaining lab banter, unforgettable conference trips, excellent peer critiques, and innumerable sake bombs, I feel quite fortunate to be a part of this great team - thanks so much!

Beyond my own lab, there are many others in academia I'd like to thank. Thanks to Paul Mitiguy for his help with AutoLev and kinematic modeling, to Eugene Mizusawa for giving me the chance to teach a team of high schoolers how to CAD and build an electric hot rod, to Sven Beiker and Markus Maurer for their helping convince me that I really *should* simply accept that cars will be my future career, and to Mike van der Loos and Judy Illes for their friendship, support, and requirement that every visit with them elicits vehicle dynamics experiences (a.k.a. fun, windy roads).



Figure 1: The Dynamic Design Lab, celebrating the success of their AAC conference on August 21, 2007

A special thanks goes to my reading and defense committees, too. Thanks to Bob Tatum, who graciously offered to join as my defense chair, chat about cars, and sign off as saying I'd passed my defense (woo-hoo!). Thanks also to Bernie Roth and Steve Rock for not only agreeing with Bob Tatum on defense day, but also providing excellent feedback and insight into my dissertation and its revision - I truly appreciate your time sorting though this tome. Finally, I wanted to thank Ed Colgate. Not only was he my undergraduate advisor at Northwestern University, but I was fortunate enough to have him on my Ph.D. defense committee. It was a great thrill to have *both* of my academic advisors there on that day, and I thank you for taking the time to fly out from Illinois and be a part of the defense and its celebration (captured in Figure 2).

While at Stanford, I had many others indirectly help me through the dissertation



Figure 2: The author (center) on the evening after passing his defense with graduate advisor Chris Gerdes (left) and undergraduate advisor Ed Colgate (right) on June 15, 2009

process. Some call this broadening my interests, while others might admit that it's actually maintaining my sanity. I'd like to thank the Stanford Wind Ensemble and its conductor, Giancarlo Aquilanti, for giving me a great musical outlet for six years. Not only they give me a fun group with whom I rehearsed and performed, but they also gave me the chance to perform with my dear friend, Yana Reznik, by inviting her to play a concerto with us and gave me the opportunity of traveling through Spain, Portugal, and Morocco on a concert tour. Grazie, Giancarlo! Also, I'd like to thank Tanya Shashko for helping me learn French, equipping me to take a post-doctoral position to do more experimental car stuff in France. Merci, Tanya! Finally, I'd like to thank Mark Cutkosky and Larry Leifer for the great experience of ME310, a globally-distributed, corporate-sponsored, open-ended, year-long design course. Truly, these experiences helped round out my tenure at Stanford - thank you all!

One story I'd like to share is that of a promise I made to myself and a large, temporary wooden structure. At Burning Man each year, they have a "temple," which is a large, beautiful, and temporary structure. It is designed to provide its visitors a common place for introspection to which they can assign meaning/non-meaning as they desire. People are encouraged to add to the temple by leaving messages or artifacts there, all of which are ceremonially burned along with the temple itself at the end of the week-long Burning Man art festival. In 2007, I went in and found a dissertation on the trajectories of land vehicles, a topic that is somewhat related to my own, and read the acknowledgements. The author thanked the organizers of Burning Man for providing this space, and was using it as the final culmination of his Ph.D. dissertation. In 2008, at a motivational low point in my Ph.D., I returned to the temple and left a message: "Dear temple, next year I will return with my Ph.D. dissertation and leave it here for you to burn. Thank you, Shad." Of course, being a temporary wooden structure in the middle of the desert, this wasn't a promise to a higher concept of any kind but rather simply a stated promise to myself to push forward and finish. Nevertheless, it worked. In 2009, after finishing the first draft of my dissertation and successfully defending it, I returned again, left my dissertation, and watched it burn along with all of the other messages, artifacts, and symbols of the other 45,000 visitors, illustrated in Figure 3. So I, in turn, would like to thank the builders of the temple for their role in helping me finish this Ph.D.



Figure 3: The author's dissertation draft being burned at Burning Man's temple on September 9, 2009

I truly have a great circle of friends that have helped me through these last several years. In particular, I'd like to thank Karlin Bark, Eric Berger, Mark Bianco, Adrian Bischoff, Sylvie Denuit, Nicola Diolaiti, Dave Dostal, Anu and Rohit Girotra, Carla and Dave Leibowitz, Julie Litzenberger, Kelsey Lynn, Megan Macdonald, Renata Melamud, Chris Milne, James Parle, Rich Prillinger, Dev Rajnarayan, Greg Reiker, Yana Reznik, Ted Rorer, Morgan Royce-Tolland, Suresh Sainath, Karen and Simon Scheffel, Philipp Skogstad, Rebecca Taylor, Nikki Willmering, and Keenan Wyrobek. You guys truly are amazing, and there are so many things I could say about each of you and how much you've supported me. I can only hope that I am able to provide as much support to you as you have to me.

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Figure 4: The author's "circle of cars" from his first car to his final Ph.D. project



Figure 5: The author (left) being hooded by Chris Gerdes (right) on June 15, 2008

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Chapter 1

Introduction

1.1 Motivation

Since the beginning of the automobile, engineers have worked continuously to increase its maneuverability. As maneuverable as today's cars have become, they have yet to reach their full potential.

One measure of a car's maneuverability is its turning ability, which is governed by lateral tire forces acting between the tire and the road. This is illustrated in Figure 1.1, where F_{yf} and F_{yr} are the lateral tire forces acting on the front and rear axles, respectively.

For conventional cars, these tire forces arise principally from slip angles. The slip angle α of a tire is the angular deviation between its heading and its velocity. When traveling in a straight line, the slip angles of a car's tires are near zero. Steering the vehicle causes a slip angle on the front axle, resulting in a front lateral tire force F_{yf} . This force induces lateral/yaw vehicle motion which results in a rear slip angle, resulting similarly in a rear lateral tire force F_{yr} .

The limits on turning ability are determined by the limits of these lateral tire forces. While tire/road friction does limit the maximum tire forces possible, a significant portion of this friction is not utilized when lateral force is generated by slip angle.

Besides slip angle, there is another way to generate lateral tire forces: camber.



Figure 1.1: Tire forces generated due to slip angle on two cars built by the author: Stanford's P1 steer-by-wire testbed at Altamont Raceway (left) and the author's Lotus Super Seven replica at Thunderhill Raceway (right)

This is the method typically employed by bicycles and motorcycles, as illustrated in Figure 1.2. Here, a camber angle γ is generated by leaning the motorcycle into the turn.

Generating lateral forces by camber instead of slip angle causes a very different set of conditions in the tire contact patch. As a result, they utilize available friction differently. This thesis shows that with camber it is conceptually possible to generate up to 30% more lateral tire force with a specialized tire design. Therefore, this thesis uses active control of camber, in coordination with active steer and suspension, to maximize lateral force capability. The result is a more maneuverable vehicle with increased turning capacity.

The goal of the active camber concept is to provide an automobile with greater turning capabilities, resulting in a better-handling and more maneuverable vehicle. The benefits of a more maneuverable automobile are numerous. For example, a more maneuverable car could allow for:

- Increased collision avoidance capabilities
- Enhanced ability to maneuver under adverse weather conditions



Figure 1.2: Tire forces generated due to camber angle: a sports motorcycle on a windy road

• Higher performance for sports and race cars

In many ways, this is an application of the control-configured vehicle (CCV) design philosophy to automobiles. A CCV design process is one in which control systems play an early and very important role [61]. This allows the effect of integrated design to be exploited to an extent not possible otherwise [43]. This is because active control can be a powerful design tool, allowing designers to relax or remove design constraints. This philosophy originated in aerospace, where control systems have opened the design space and made possible aircraft that are faster, lighter, more efficient, and more maneuverable [44].

One application of CCV is maneuver flaps, which are control surfaces that significantly increase the lift capabilities of the wings, enabling notably higher turn rates. The maximum turn rate is one thing that quantifies the maneuverability of an aircraft [3] [50]. In some highly-maneuverable fighters, designers integrate maneuver flap actuation into the control of the aircraft [55]. By coordinating maneuver flaps with the other control surfaces, the pilot's ability to command high turn rates is greatly increased [54].

Active camber represents additional actuation that, when included in the control of an automobile, increases its turning ability. As such, it is analogous in concept to actively-controlled maneuver flaps. The results are also similar: a marked increase in maneuverability.

1.1.1 Automotive Maneuverability

Similar to aircraft, one measure of an automobile's maneuverability is its turning ability. This is, in turn, characterized by its yaw rate response. The turning ability of an automobile can be characterized by:

- Yaw rate response time, stability, and smoothness. These are measures of the transient response.
- Maximum achievable yaw rate. This is a measure of the steady-state response.

Many control systems have been developed that alter a vehicle's maneuverability by addressing the transient yaw rate response. For example, direct yaw moment control systems (also known as electronic stability or vehicle dynamics control systems) use differential braking to ensure stability of the yaw rate responses [59] [51]. These systems may actually decrease the maximum achievable yaw rate slightly to accomplish this goal, in effect trading maximum maneuverability for stability. Other examples can be found in the development of four-wheel steering controllers. Here, control over the rear wheels provides, for different operating conditions, the ability to reduce the response time or provide damping to the yaw rate response by providing direct control over the rear slip angle [13] [60] [1].

However, none of these make significant increases to the maximum steady-state yaw rate. Although the above controllers are very successful at shaping the transient response of and extending stability of the yaw rate response, none enhance maneuverability by increasing the maximum yaw rate possible.

By using specialized tires that allow greater utilization of friction with camber and then using control systems to coordinate the driver's turning command by both steering and cambering all four wheels independently, the steady-state yaw rate of the vehicle can be altered significantly. Results of this thesis predict a potential gain of 20-30%. The result is a vehicle with extreme maneuverability.

1.1.2 The Case for Active Camber

To understand what limits a vehicle's maneuverability, it's instructive to consider the simple, planar vehicle model illustrated in Figure 1.3. This vehicle has wheelbase L with distances from each axle to the vehicle CG given by a and b. To provide cornering, lateral forces F_{yf} and F_{yr} are applied to the front and rear axles, respectively. Its equations of motion are given by:

$$I_z \dot{r} = aF_{yf} - bF_{yr} \tag{1.1}$$

$$ma_y = F_{yf} + F_{yr} \tag{1.2}$$

Where I_z is the yaw moment of inertia, m is the vehicle mass, r is the yaw rate, and a_y is the lateral acceleration. In steady-state, $\dot{r} = 0$.



Figure 1.3: A simple car model

The fundamental limit of a vehicle's steady-state turning ability is determined by tire forces. With a given friction surface, they can only generate so much lateral force. Usually, this is approximately proportional to the normal load on that tire, given by:

$$F_{yf,max} = \mu \frac{b}{L} mg \tag{1.3}$$

$$F_{yr,max} = \mu \frac{a}{L} mg \tag{1.4}$$

where μ is the effective coefficient of friction between the tire and the road and g is

gravitational acceleration. By substituting Equations 1.3 and 1.4 into Equation 1.2, a maximum lateral acceleration $a_{y,max}$ is found:

$$a_{y,max} = \mu g \tag{1.5}$$

In steady state, lateral acceleration a_y , yaw rate r, and vehicle speed V are related by:

$$a_y = rV \tag{1.6}$$

Therefore, given any type of steering control, the steady-state turning capability of the vehicle is still limited by μ .

The opportunity of active camber lies in the fact that by actuating a tire only by generating slip angles, much of the adhesion friction in the contact patch is *not* utilized. This can be understood by taking a closer look at how tire forces are generated in the tire contact patch, illustrated below using a simple tire model known as a 1D tire brush model.

When a tire is free rolling, it tends to travel straight ahead in the direction it is pointing. But, when a tire is subject to a slip angle α to generate lateral tire force F_y , it no longer follows this path, illustrated in Figure 1.4.

Friction between the rubber and the road cause the tire contact patch to deform in response to this slip angle, shown in Figure 1.5. The concept is that there are small "brush" elements on the bottom of the contact patch. The base of the brushes are attached to the tire and the tips adhere to the road by friction. Lateral tire forces F_y are generated by the restoring forces caused by their lateral deformation. As a brush element of the tire rolls into the contact patch, its lateral deformation is initially zero at the leading edge (on the right). For free rolling, the brush continues to the rear of the contact patch without appreciable deformation, yielding no lateral tire force. But, when moving at a slip angle α , the deformation increases linearly as it travels toward the trailing edge, returning to zero deformation beyond the trailing edge where the rubber and road are no longer in contact. As shown in the figure, the restoring force associated with this deformation induces a triangular force distribution for F_y .



Figure 1.4: Diagram of tire in free rolling (top) and at a slip angle (bottom)

This deformation profile represents the *force demand* in the contact patch. The *force availability* is the available friction in the contact patch, determined by the vertical pressure distribution. Figure 1.6 represents the longitudinal variation of vertical force distribution by a parabola, similar to many tire models in the literature [39] [14]. Vertical pressure is highest in the middle of the tire and tends toward zero at the leading and trailing edges. Illustrated are the resulting adhesion and sliding friction limits imposed by this vertical pressure distribution.

At the rear of the contact patch, the available adhesion friction will limit the actual tire force generated. When adhesion is exceeded, that part of the contact patch will be in sliding. As slip angle is increased, the point of sliding moves forward, causing more and more of the contact patch to exceed its adhesion limit and slide. This is illustrated in Figure 1.7, with the force demand by slip angle increasing from left to right. It is clear that a large portion of the available adhesion friction is not being utilized by slip angle. This results in a characteristic tire curve relating slip angle α to lateral force F_y , given in Figure 1.8.

When a tire is cambered, it causes a very different type of deformation. Shown in Figure 1.9, operating the tire at a camber angle γ (left) causes the tire to deform laterally into an arc (right).



Figure 1.5: Diagram of tire contact patch in free rolling (top) and at a slip angle (bottom)



Figure 1.6: A 1D, parabolic model of contact patch vertical force distribution (left), yielding adhesion and sliding friction limits (right)



Figure 1.7: Lateral force distribution for slip angle using a 1D tire brush model



Figure 1.8: A typical tire curve, mapping slip angle α to lateral force F_y



Figure 1.9: Diagram of a tire at camber angle γ (left), causing an arc-like lateral deformation (pictured right)

This causes the base of the brushes to follow an arc-like trajectory. Since the tips of the brushes follow the trajectory of the road, which is relatively straight, this causes an overall arc-like lateral deformation profile as shown in Figure 1.10.



Figure 1.10: Diagram of tire contact patch in camber

This causes a *force demand* that is a much closer match to the available friction. Figure 1.11 illustrates this for increasing camber demand from left to right. While there are some other effects not captured by this model that don't allow *all* of the adhesion friction to be utilized by camber, the basic result is still the same: camber has the ability to utilize more of the available friction in the contact patch than slip angle. Therefore, to attain the highest possible lateral force, the tire should be actuated using camber with zero effective slip angle.



Figure 1.11: Lateral force distribution for camber using a 1D tire brush model
1.1.3 Prior Art

There has long been discussion about cambering vehicles, but until recently there was little development. One early example is the 1967 Milliken MX1 "Camber Car," developed by Milliken Racing Associates [32] and pictured in Figure 1.12. Its purpose was to examine the idea of using a static camber angle to increase the lateral force capability using motorcycle tires. It did not actively control camber.



Figure 1.12: 1967 Milliken MX1 Camber Car

Perhaps the most important previous example of active camber work is the 2002 Mercedes F400 Carving Concept [2], pictured in Figure 1.13. The purpose of the F400 was to showcase a multitude of new technologies being developed at Mercedes, and one of these concepts was active camber. The engineers at Mercedes were motivated by the same principle as this thesis: camber holds the potential for better friction utilization.

For this concept, Pirelli developed a special tire, shown in Figure 1.14. This tire was flat on the outside and rounded on the inside. This gave the tires the ability to generate lateral forces by cambering the outer wheels in a turn by 20°. Since the tires were only rounded on one side, they could not camber both directions.

The engineers developed the suspension first as a conventional, non-cambered suspension using conventional design criteria. Then, camber actuation was superimposed on top of this by using hydraulic actuation on the steering knuckle to push the bottom of the tire outward. This accomplished 20° of camber in one direction, and is shown in Figure 1.15.

The Mercedes engineers intended this vehicle primarily as a proof-of-concept. As such, this concept was very successful: it demonstrated a 28% gain in peak lateral force capability. This is similar to the result from tire model developed in this thesis, giving validity to its prediction.



Figure 1.13: 2002 Mercedes F400 Carving Concept

This thesis is not the first to examine the idea of using camber to increase tire forces. However, it is first to develop an *integrated* approach to suspension and tire development for maximizing lateral tire force using active steer, vertical suspension (jounce/rebound), and camber. By integrating all of these elements from the start, it is possible to explore a larger design space.



Figure 1.14: Pirelli tires for the 2002 Mercedes F400 Carving Concept



Figure 1.15: Suspension of 2002 Mercedes F400 Carving Concept

1.2 Overview and Background

The contributions of this thesis follow two main threads:

- Active camber tires. Passenger car tires are not well-suited for use at high camber angles. Although the simple tire model developed in Section 1.1.2 predicts a large gain in peak lateral force by using camber as opposed to slip angle, motorcycle tires do not exhibit this characteristic (see Section 1.2.1). To exploit fully the benefits of cambering, the active camber concept requires new, specialized tires. To accomplish this, a new, 2D variant of a brush tire model is developed that considers the force distribution in the contact patch in both lateral and longitudinal directions. This gives a more complete picture of how camber forces are generated than existing models, which typically ignore the lateral distribution. Similar to other brush models, this requires a model of the vertical force distribution in the contact patch. Several contact patch model. The resulting 2D brush tire model is used to characterize existing motorcycle tires and to predict what design parameters need to be altered to generate a tire that *does* exploit fully the benefits of cambering.
- Active camber suspension system. Of course, a conventional suspension system will not suffice for the active camber concept: a simple mechanism connected to a steering wheel will not be sufficient to coordinate the steer and camber angles of all four wheels to maximize lateral force. A specialized, mechatronic suspension system is required that provides full control over the tire to maximize maneuverability. However, the design *criteria* presented by existing suspension design literature do not address active camber. Therefore, this thesis articulates a clear set of design *principles* rooted in the design of mechatronic systems and applied to a kinematic model of the suspension system. By using the forward kinematics, inverse kinematics, and Jacobians of the model, the design principles are mapped into design criteria are developed that are similar

to existing suspension design literature. By applying this process to a suspension system with active camber, active steer, and active vertical suspension, design criteria for the active camber concept are developed. These are then used to guide the design and construction of a prototype suspension system, illustrated in Figure 1.16.



Figure 1.16: Prototype active camber suspension system

The prototype suspension system is attached to chassis dyno rollers, providing an experimental rolling road for testing. This is used to measure the performance of three different motorcycle tires. Not only does this data serve to demonstrate the capability of the suspension system as a research testbed; it also serves to validate the tire model.

1.2.1 Active Camber Tires

To allow use at high camber angles, the active camber concept requires tires with a curved profile, more similar to that of a motorcycle than a car. However, motorcycle tires themselves will not suffice - they do not exhibit large gains in peak lateral force by using camber as opposed to slip angle. This is in part because the design of motorcycle tires is constrained. The basic physics of a motorcycle dictate the approximate force needed from camber.



Figure 1.17: Free-body diagram of motorcycle turning with camber and tire forces

Consider the free-body diagram of the motorcycle pictured in Fugure 1.17. To maintain a steady-state camber angle and therefore sustain a turn, the lateral force F_y , vertical force F_z , and lean angle ϕ of the motorcycle are fixed to one another:

$$\frac{F_y}{F_z} = -\tan(\phi) \tag{1.7}$$

Typically, the force from camber is slightly less than this value. The remainder of the force is generated by slip angle (See Chapter 3). The end result is that $\geq 45^{\circ}$ of camber are needed to get $\frac{F_y}{F_z} \geq 1$.

At high camber angles, the inside of the tire is moving over the road *slower* than the center, and the outside of the tire *faster* than the center. As a brush on the inside of the tire enters the contact patch on the leading edge, the brush tip is pulled rearward at a rate faster than the brush base, which is attached to the tire body. This causes a forward *longitudinal* deformation of the brush element. Likewise, on the outside, a rearward longitudinal deformation results.

Similar to lateral deformations caused by slip angle α and camber angle γ , the restoring forces induced by this deformation generate longitudinal tire forces. However, the contributions to longitudinal force of the inside and outside halves of the tire are opposite in direction and therefore cancel - no net longitudinal force is generated, but some friction capability is used to maintain this longitudinal deformation. This is the primary effect that is not captured by the 1D brush model outlined in Section 1.1.2, and reduces the available friction for lateral tire force generation. This effect is most pronounced for tires which require extremely high camber angles, like those of a motorcycle.

Therefore, to exploit fully the benefits of cambering, the active camber concept requires new, specialized tires. These tires have properties aimed at reducing longitudinal deformation, increasing friction utilization. As a side benefit, reducing the camber angle γ required to attain high lateral force F_y simplifies suspension design.

To understand the design and expected performance of these specialized tires, this thesis:

- Develops a new variant of a brush tire model. The goal of this model is to be complicated enough to capture the effects of curved-profile tires and steadystate camber force generation, but simple enough to facilitate interpretation and inclusion into vehicle handling models.
- Validates the tire model using motorcycle tires. Although the parameterizations are different, the mechanisms of camber force generation and basic tire geometries are similar for motorcycle tires and the specialized tires. Therefore, measurements of existing motorcycle tires are used to validate the model.
- Hypothesizes the design of a specialized tire for the active camber concept. Based on the validated tire model, a specialized tire design is hypothesized that could extract 20-30% more lateral force by using camber as opposed to slip angle, similar to the results of the Mercedes F400.

Tire Model Survey

Broadly-speaking, there are three main categories of existing tire models. The first is empirically-based models. By far, the most widely used empirical tire model used today is the "Magic Tire Formula" by Pacejka [40] [39]. It has been applied not only to passenger car tires but also motorcycle tires [8] [9]. As its name implies, it does an excellent job of matching experimental data. Therefore, it is used very commonly in vehicle handling studies. In fact, some studies of more complicated tire models use it in lieu of experimental data as the datum for comparison [10] [42]. However, as its name unabashedly admits, it also lacks a physically-based explanation. Therefore, it is unsuitable for hypothesizing a new tire design.

On the opposite end of the spectrum, the second category of tire models are the finite-element-based models. Tires have been studied in the context of finite element modeling since the early 1980s [52] [58], and have become increasingly more refined since [22] [25]. They are particularly effective at modeling transients and highfrequency noise and vibration characteristics [63] [42], and have also been applied to cambering [62] and motorcycle tires [24]. However, the complexity of these models limits their applicability. More recent work by Gipser [17] is aimed at reducing the complexity of these models and making them useable to a broader user base, resulting in a package for ADAMS finite element software. However, due to their complexity, finite element models are not typically used in vehicle handling studies.

The third category of tire models are simple, physically-based models. The most common class of models here are brush (or brush-and-ring) tire models. Several different versions of brush models have been used since the 1940s [21] [39]. Due their simplicity, small parameter count, and reasonable accuracy (albiet less than the other two categories), they are commonly used in vehicle handling studies. Therefore, this thesis develops a brush tire model for use with curved-profile tires and camber.

Brush Tire Models

The basic schematic of a brush model is illustrated in Figure 1.18 for a 1D case, similar to the 1D brush model used in 1.1.2. The following breakdown of modeling



Figure 1.18: Diagram of a 1D tire brush model: a side view of the whole tire (top) and a close-up, bottom view of the brushes and band in the contact patch (bottom)

steps in a brush model describe its components:

- Contact patch shape and vertical pressure distribution. These are typically used as "inputs" to the brush tire model and characterize the *force availability* in the contact patch.
- Brush elements and horizontal stress distribution. Inducing onto the tire a slip angle α , camber angle γ , and/or longitudinal slip κ (from driving/braking a wheel, see Chapter 3) cause a mismatch between the velocities of the tire body and road, known as slip velocities. Small, independent brush elements in the contact patch which are attached to the tire body at their base and adhered to the road at their tip deform as a result of these slip velocities. Because the brushes have finite stiffness, the deformation profile of the brush elements leads to a horizontal stress distribution. This characterizes the *force demand* in the contact patch.
- Friction model. Once the force demand is calculated, it is compared to the force availability determined by the vertical pressure distribution, enforcing a limit due to friction. If the force demand of a brush element is at or below the adhesion limit characterized by an adhesion coefficient μ_a , the brush remains in adhesion. If this limit is exceeded, the brush is said to be sliding and a sliding friction model determines the resulting limited force from the brush element.
- Tire band/ring and carcass model. A thin band or ring serves to connect the brush elements to one other, and is often modeled as a rigid ring, an elastic beam, or an elastic string. This band/ring is, in turn, connected to the rigid wheel or hub by the tire carcass, which is often modeled as either rigid or compliant. Models that assume one or both of these are not rigid require some iteration to solve because compliance/elasticity alters the location of the bases of the brush elements. Tire carcass compliance causes a bulk movement where all brush elements are moved the same way, whereas band elasticity can cause each brush element to move differently.

Table 1.1 presents a survey of many different types of brush models, all of which use a different subset of assumptions about model components. Most brush models, including the first six in the table, were developed for use with passenger car tires. All of these are 1D and assume uniformity in the lateral direction of contact patch shape, vertical pressure distribution, and horizontal stress distribution. As such, they can be represented using a single row of longitudinal brushes, shown in Figure 1.18.

Analytically, parabolic and uniform pressure distributions are the simplest and are therefore used frequently. However, data indicates that the actual pressure distributions tend to be somewhere between parabolic and uniform, resulting in other model choices. Different ring/band and carcass model choices have the primary effect of refining the prediction of tire moments and transients.

Different choices of sliding friction model result in different predictions about the shape of the tire curve (see Figure 1.8) near its peak friction. When the sliding friction coefficient μ_s is assumed equal to the adhesion coefficient μ_a , the "roll off" characteristic at high slip angles disappears: high slip angles do not decrease the tire force. Sliding friction models that have a dissimilar coefficient (with or without a gradual transition from adhesion to sliding) can capture the roll off characteristic typically exhibited by tires. More complicated friction models, such as the LuGre model [10], can model friction at high slip values even more accurately.

Brush models that are intended for application at large camber angles are much less common. Four are given in Table 1.1, all of which are variations on the 1D brush model representation. Because of this, they have difficulty capturing the longitudinal deformation due to camber and the resulting twisting moment in the contact patch about the z-axis. Meijaard [31] and Goel [18] still assume a 1D tire representation and therefore does not capture these effects at all. Fujioka [15] assumes an elliptical contact patch, but still assumes uniformity of vertical pressure and horizontal stress in the lateral direction; however, by using an elastic band model he can explain a similar twisting moment (albiet not arising from longitudinal deformation). Pacejka [39] considers two rows of 1D brush elements: one on the inside and one on the outside of the tire. This model *does* exhibit longitudinal deformation due to camber and the resulting twisting moment, but not in a manner that accurately depicts the shape or vertical pressure distribution of a curved-profile tire.

Therefore, the brush model in this thesis uses a 2D representation of the brush model, depicted in Figure 1.19. Here, the contact patch shape is 2D ellipse and the vertical pressure distribution is represented in 2D, removing the restriction that it must be uniform in the lateral direction. Similar to car tires, motorcycle tire pressure distributions are often somewhere between parabolic and uniform, which is represented by a 1-parameter polynomial for each direction (longitudinal and lateral). This results in a much closer representation of measured contact patches (see Chapter 2).

Because the contact patch representation is 2D, it is given a 2D array of brush elements and corresponding 2D horizontal stress distribution. This has the key benefit of depicting the longitudinal deformation due to camber more accurately than 1D brush model implementations. To complete the model, a simple sliding friction model similar to Sakai [45] is used and the tire ring/band is assumed rigid, although bulk deformation of the tire contact patch is modeled by a compliant tire carcass.

The resulting model is validated against experimental data taken with three different motorcycle tires, showing good fit. Matching expectation, the parameters used to characterize these motorcycle tires predict no appreciable gain in peak lateral force by using camber instead of slip angle. The tire model is then used to hypothesize the design of a specialized tire for the active camber concept that exhibits an increase of 20-30% more lateral force by using camber, similar to the results of the Mercedes F400.



Figure 1.19: Diagram of a 2D tire brush model for a toroidal tire: a side view of the whole tire (top) and a close-up, bottom view of the brushes and band in the contact patch (bottom)

	contact	patch	brush	es	tire	body (applied
-		vertical	horizontal		band/		to large
model		pressure	stress	sliding	ring	carcass	camber
author	shape	distribution	distribution	friction	type	type	angles
Fromm $[21]$	rectangle	1D parabolic	1D line	$\mu_s = \mu_a$	rigid	rigid	no
Fiala [14]	rectangle	1D parabolic	1D line	$\mu_s = \mu_a$	elastic	compliant	no
Dugoff [12]	rectangle	1D uniform	1D line	$\mu_s=\mu_a$	rigid	rigid	no
Bernard [4]	rectangle	1D trapezoidal	1D line	$\mu_s \neq \mu_a$ gradual transition	rigid	rigid	no
Sakai [45] [46] [47] [48]	rectangle	1D exponential	1D line	$\mu_s \neq \mu_a$ gradual transition	elastic	compliant	no
Deur [10]	rectangle	asymmetric 1D trapezoidal	1D line	LuGre dynamic model	rigid	rigid	no
Meijaard [31]	rectangle	modified 1D parabolic	1D line	$\begin{array}{c} \mu_s \neq \mu_a \\ \text{abrupt} \\ \text{transition} \end{array}$	elastic	rigid	yes
Goel [18]	rectangle with rounded edges	1D trapezoidal	1D line	$\mu_s = \mu_a$	rigid	rigid	yes
Fujioka [15]	ellipse	1D parabolic	1D line	$\mu_s=\mu_a$	elastic	rigid	yes
Pacejka [39]	rectangle	1D parabolic	two 1D lines	$\mu_s=\mu_a$	elastic	rigid	yes
Laws	ellipse	2D polynomial	2D surface	$\begin{array}{c} \mu_s \neq \mu_a \\ \text{gradual} \\ \text{transition} \end{array}$	rigid	compliant	yes

CHAPTER 1. INTRODUCTION

Table 1.1: Survey of brush tire models

1.2.2 Active Camber Suspension System

The active camber concept requires a specialized, mechatronic suspension system that provides full control over the tire to maximize maneuverability. To do this, the suspension requires active steer to control slip angle α , active camber to control camber angle γ , and active suspension to control the vertical load F_z . The suspension should be designed for use on a full-sized, modular test car (see Section 4.2.1).

Automotive suspension design is an established field with many well-understood concepts and design criteria [33] [11] [34]. Existing literature has examined some of these design criteria in the context of active suspension [49] [23]. Furthermore, previous work by the author applies some of these criteria to active steer, or steer-by-wire, systems [27] [5]. However, existing literature does not explain how these criteria should be interpreted or even defined for a suspension with active camber.

One key suspension design concept is roll center. It is usually derived from a simplified, 2D schematic of the suspension system [33] [11] [34] [41] [16], shown in Figure 1.20. Here, a forward-facing view of one axle's suspension systems is represented, with the suspension members that connect the wheel to the vehicle and control suspension motion represented as simple links. When lateral tire forces are applied to the tire, they induce reaction forces in these links (top of Figure 1.20). Roll center abstracts the effects of these reaction forces and can be used to predict and tune vehicle roll motion, as well as predict rollover stability [20] [36].

The middle of Figure 1.20 illustrates how the roll center is typically constructed. First, the instantaneous centers of each wheel's suspension members are found. Then, lines are drawn from these instantaneous centers to the centers of their respective tire contact patches. The roll center for each wheel is located along these lines, directly under the vehicle CG. Since the suspension systems in Figure 1.20 are symmetric, the two roll centers are coincident, located at point RC.

The interpretation of roll center is illustrated in the bottom of Figure 1.20. Here, the roll center is used as an effective force application point. The effects of reaction forces in the suspension members are abstracted by applying the lateral tire force F_y directly to the vehicle body at the roll center RC. The design criteria resulting from this abstraction is the height of the roll center above the ground h_{rc} . This can be



Figure 1.20: Diagram of the conventional roll center concept



used to model many aspects of vehicle roll motion [28] [6] [20] [37].

Figure 1.21: Simplified, 2D schematic of an active camber suspension system

A simple, 2D representation of a suspension with camber actuation is shown in Figure 1.21. Camber actuation is accomplished by pivoting the additional link. When this additional link is added, the conventional roll center construction is rendered in-applicable. While previous work has addressed the limitation imposed by defining roll center by a 2D construction [38] [35] [53] [56], none have extended it to a suspension with active camber. Without further analysis, the concept of roll center is lost.

To understand how conventional, established suspension concepts and design criteria should be extended to mechatronic suspensions, it is instructive to take a step back and examine their underlying meaning. Concepts such as roll center can be viewed as a way to map fundamental design *principles* to calculable design *criteria*. This is illustrated in Figure 1.22, where the top arc represents mapping concepts used in conventional suspension design (including, but not limited to, roll center, pitch center, and steer axis). One difficulty of extending these concepts to mechatronic suspension systems is that these concepts are usually presented as disconnected ideas, often lacking explicit connection to the design principles from which they derive.

Therefore, to extend the body of existing suspension design criteria to mechatronic suspension systems, the first step is to state explicitly these design principles. The method used in Chapter 4 states these in terms of control and estimation objectives such as decoupling, disturbance rejection, and stability. These principles are applied to a full, 3D kinematic model of the suspension system. Then, these principles can



Figure 1.22: Mapping of design principles to design criteria

be mapped to design criteria by using the forward kinematics, inverse kinematics, and Jacobians of the suspension system. This is illustrated by the bottom arc of Figure 1.22.

By applying this process to conventional suspension systems, many existing suspension design criteria can be derived (see Chapter 4). Applying this process to an active camber suspension system allows interpretation of these existing design criteria in the context of active camber and generation of new design criteria specific to active camber.

Many existing suspension design criteria can be derived by applying this process to conventional suspension systems (see Chapter 4). Therefore, this thesis applies it to an active camber suspension system. This allows interpretation of existing design criteria in context of active camber, and generates new design criteria specific to active camber.

For example, consider again roll center. Figure 1.23 illustrates the roll center RC along with a vertical suspension actuator, represented here by a spring. Reacting

applied lateral force F_y induces a vertical reaction force F_{zr} applied by the suspension members to the vehicle body. This causes a change in the vertical force reacted by the suspension actuator F_{zs} , given by:

$$F_{zs} = F_z - F_{zr} \tag{1.8}$$



Figure 1.23: Diagram of the roll center concept using forces

For conventional suspension systems, this actuator is simply a passive spring. Therefore, a change in the actuator force is a chance in the spring force. This induces change in the spring position, which in turn causes vehicle roll motion. This is what leads to the name of "roll center."

For active suspension systems, the vertical suspension actuator is actively controlled. There is no longer a 1:1 link between lateral tire force and roll motion. However, there still is a link between lateral tire force F_y and change in suspension actuator force F_{zs} . Therefore, the roll center is used as a measure of *coupling* between lateral and vertical tire control. Rather than use it to specify vehicle roll motion, it can be used to *decouple* vertical and lateral tire control. This is accomplished by setting the roll center height h_{rc} at zero for the nominal suspension configuration, effectively placing it on the ground.

In general, roll center height h_{rc} varies as the suspension is moved up and down. Variation in vertical suspension position is imposed by road irregularities and is one of the primary disturbance sources for suspension systems. The analysis of many simple roll models used in vehicle dynamics studies show that it is the *distance* between the roll center RC and the vehicle CG that is important to the roll mode [28] [41] [7] [6] [20]. Therefore, for disturbance rejection, the roll center height h_{rc} should vary with vertical suspension position in a way that keeps the distance between the roll center RC and vehicle CG constant.

Because it is a coupling term, the roll center height h_{rc} can be expressed using the Jacobian of the suspension system model. For conventional suspension systems, the resulting h_{rc} is the same. The difference is that now it can be expressed for a broader class of suspension systems, including those with active camber (presented by author in [29]).

Chapter 4 outlines in detail the design principles, mappings, and resulting design criteria that result from this process. Then, these are used to design the prototype active camber suspension system, detailed in Chapter 5. One major challenge to suspension design is packaging. As a result, the process of going from desired design criteria to a physical suspension design is not straightforward and often requires some degree of design compromise: it is usually not possible to make a design that is optimal for every design criteria.

The challenge of packaging for the active camber suspension is further exacerbated by the need to allow a large range of camber articulation and fit three large actuators. A mechanism on the order of 1 m³ in volume needs to provide 75° of camber movement, 30° of steer movement, and 100 mm of vertical suspension movement while rigidly supporting tire forces of up to 8000 N in each direction. One process for tuning design criteria while navigating through packaging constraints to develop physical suspension design parameters is developed in Chapter 5.

This process is used to determine the final suspension design, which is then fabricated and attached to chassis dyno rollers (see Section 5.3.3). These rollers provide an experimental rolling road for testing, which is used to measure the performance of three different motorcycle tires. Not only does this data serve to demonstrate the capability of the suspension system as a research testbed; it also serves to validate the tire model.

1.3 Thesis Contributions and Outline

The contributions of this thesis are aimed at the realization of the active camber concept for extreme maneuverability. Based on the overview in Section 1.2, this thesis:

- Develops and validates a new model for the 2D shape and vertical pressure distribution of the tire contact patch (Chapter 2). When a curved-profile tire is used to allow high camber angles, similar to existing motorcycle tires, the resulting contact patch shape and vertical pressure distribution requires a 2D representation. This is in contrast to passenger car tires, whose near-rectangular shape and nearly-uniform lateral distribution allow for a reasonable approximation in 1D. To illustrate this, contact patch measurements for three different motorcycle tires at different operating conditions are presented. These are measured using pressure-sensitive paper while loading the tires at different camber angles, inflation pressures, and normal loads on both a flat surface (similar to a road) and a drum (similar to many tire testers, including the prototype active camber suspension system). Observations of these measurements, as well as the derivation of two physically-based models based on common assumptions in previous brush model work, lead to the development of a new, semi-empirical contact patch model. This model is used to parameterize the three tires, successfully capturing the shape and vertical pressure distribution in all cases when the contact patch is not sufficiently distorted by the sidewall. Note that sidewall distortion generally causes degredation of tire performance and high-performance use typically avoids this occurrence.
- Extends a brush tire model to 2D to capture the effects of cambering tires (Chapter 3). Brush tire models are a class of tire force models commonly used in vehicle handling models. Typically, these models are 1D and only consider pressure distribution variations in the longitudinal direction of the contact patch. While sufficient for conventional passenger car tires with slip angles, this is not a good approximation of a curved-profile tire with camber. Therefore, this

thesis develops a 2D brush model that considers longitudinal and lateral pressure variation in both longitudinal and lateral directions of the contact patch. The contact patch and vertical pressure distribution are taken from Chapter 2. These brushes are affixed to a rigid tire carcass, whose connection to the rigid wheel is considered compliant. Experimental data is presented from three different motorcycle tires, the same as used for contact patch measurements in Chapter 2. These data result from using the prototype suspension system on an experimental rolling road, and serve to validate the 2D brush tire model. Once validated, the model is used to give a clearer picture of how camber can utilize friction better than slip angle, and is used to develop a hypothesis of how a specialized tire for the active camber concept could provide 30% more peak lateral force from camber. Furthermore, this peak can occur at a lower camber angle $(20^{\circ}-25^{\circ})$ than the peak of a motorcycle tire $(> 40^{\circ})$, requiring less actuation range and simplifying suspension design.

- Presents a set of design principles and design criteria for mechatronic suspension systems (Chapter 4). These principles are stated by using control and estimation objectives and applied to a complete, kinematic model of the suspension system. From this, design criteria are derived from the forward kinematics, inverse kinematics, and Jacobians of the suspension system. By applying them to conventional suspension systems, design criteria are developed that are similar to existing suspension design literature. By applying them to a suspension system with active camber, active steer, and active vertical suspension, design criteria for the active camber concept are developed.
- Realizes a complete prototype suspension system for the active camber concept (Chapter 5). A prototype suspension system for the active camber concept is developed using the design criteria developed in Chapter 4 while also negotiating other constraints imposed primarily by packaging. To do so, the thesis presents one method for stepping through the process of suspension design and analysis. The resulting design criteria of the prototype suspension system are presented and discussed. The final design was constructed

and implemented successfully, and was attached to chassis dyno rollers. This provides an experimental rolling road for testing, demonstrating the capability of the suspension system as a research testbed. A discussion of the observed performance of the prototype suspension system is included.

Chapter 2

Tire Contact Patch

As discussed in Section 1.2.1, one key component of a tire brush model is the contact patch shape and vertical pressure distribution. The primary purpose of this chapter is to develop a semi-empirical, 2D model for the contact patch and use it to parameterize measurements taken from three different tires.

Section 2.2 presents measurements taken from three different motorcycle tires under different loading conditions using pressure-sensitive paper. These measurements provide data on the shape, size, and vertical pressure distribution of contact patches, and motivate the contact patch model developed in Section 2.3. First, two physicallybased contact patch models are developed. Then, a semi-empirical model is presented that blends these two models (the two physically-based models are, in fact, limiting cases of the semi-empirical model).

Then, these contact patches are analyzed and parameterized using the model of Section 2.3 in Section 2.4. The model successfully captures the effects of different vertical loads, different inflation pressures, traveling on a flat surface (similar to a road) versus a drum (such as the one used in Section 5.3.3 on the suspension prototype), and camber angles up until the sidewall enters into and distorts the contact patch. Once this happens, the contact patch geometry becomes greatly distorted and tire performance degrades. At these extremely high camber angles, the contact patch model is no longer able to characterize its geometry. However, since this is not the preferred operating region for high-performance, high-maneuverability use, this is not a major modeling limitation.

For active camber suspension systems, the tire shape of interest is a toroid, similar to that of a motorcycle tire. This means that the tread profile has a constant curvature, providing consistent contact patch shape over a large range of camber angles. A schematic of the toroid is given in Figure 2.1. The tire effective overall radius is r_{te} and the tread profile radius is r_{tt} .



Figure 2.1: Diagram of tire toroid

Also included in Section 2.5 are measurements from a car tire. It is provided as an example of car tires, which are nearly uniform in the lateral direction making their geometry more similar to a cylinder than a toroid. These measurements show that the contact patch becomes distorted even when relatively small camber angles are applied, illustrating their inability to be used as an effective tire for highly-cambered suspension systems. A modified version of the semi-empirical model of Section 2.3 is developed and used to analyze these measurements, providing a comparison to the analysis of the three motorcycle tires.

2.1 Description of Tires

Table 2.1 provides the basic specifications of the three motorcycle tires and one car tire used in this chapter. Included in this table are the tire effective overall radius r_{te} and the tread profile radius r_{tt} (see Figure 2.1).

The Avon and Metzeler tires are large motorcycle tires designed primarily for the rear of custom "chopper" motorcycles, pictured in Figure 2.2. They are capable of sustaining large vertical loads, similar to those required for an active camber prototype with the size and weight of a full-sized car (about 1500-2000kg total mass). Each of the tires has a very different tire carcass design. The Metzeler ME880 tire has 0° radial steel belts and a single bias ply. The Avon Cobra tire has no steel belts and several bias plies.



Figure 2.2: Pictures of the large 300/35R18 motorcycle tires by Avon (left in both) and Metzeler (right in both)

The Dunlop tire is designed for the rear of high-performance sports motorcycles, pictured in Figure 2.3. They are not designed to handle the same amount of vertical load as the other two motorcycle tires, but its different design intent makes it an interesting comparison against the Avon and Metzeler.

Finally, the Hoosier tire pictured in Figure 2.4 is the car tire used in Section 2.5 and is designed for use as a racing slick. Note also that many tire models, including



Figure 2.3: Pictures of the 180/55R17 motorcycle tire by Dunlop

the one developed in this thesis, neglect the effects of tread elements. Therefore, it is interesting to observe measurements from this slick, treadless tire.



Figure 2.4: Pictures of the 225/45R17 racing slick by Hoosier

The Avon and Metzeler tires are also used to develop the active camber suspension prototype as described in Chapter 5. Additionally, all three motorcycle tires are used to take measurements of tire performance, as described in Chapter 3.

					overall	tread
	tire	tire	tire	rim	radius	radius
type	brand	model	size	size	$r_{te} (\mathrm{mm})$	$r_{tt} (\mathrm{mm})$
MC	Dunlop	Sportmax Qualifier	180/55R17	17x5.5	312	105
MC	Avon	Cobra AV72	300/35R18	18x10.5	325	200
MC	Metzeler	ME880 XXL	300/35R18	18x10.5	325	200
Car	Hoosier	A6	225/45 R17	17x8.0	312	(n/a)

Table 2.1: Table of tire sizes for the three motorcycle (MC) tires and one car tire used for experimentation

2.2 Contact Patch Measurements

This section presents a series of contact patch measurements taken using pressuresensitive paper and provide data on contact patch shapes and vertical force distributions. These data were collected at different camber angles γ , vertical loads F_z , and inflation pressures P.

The tire is installed into a fixture, shown in Figure 2.5. Two pressure-sensitive papers with different pressure ranges are placed on a flat platform under the tire. The hydraulic jack lifts the platform and applies a vertical load, which is measured by a load cell in the platform. The camber angle of the tire is varied for different tests. Also, a curved insert can be placed atop the flat platform for some tests to emulate a $\phi 1.2$ m drum, similar to the chassis dyno rollers described in Section 5.3.3. Finally, the measurements from each pair of pressure-sensitive papers are fused together in post-processing to form the final result.

The motorcycle contact patch figures in this section are presented as 2D plots using color to indicate the vertical pressure $\sigma_z(x, y)$. All of these are oriented with the leading edge of the contact patch to the right of the figure. The car contact patch figures in Section 2.5.2 follow the same convention.

Nominal Load and Inflation Pressure, Zero Camber Angle

Figure 2.7 shows the contact patch of the Metzeler 300/35R18 tire at zero camber on a flat surface with a nominal load of $F_z = 3000$ N. The same conditions are shown



Figure 2.5: The fixture used to take contact patch measurements with pressuresensitive paper

for the Avon 300/35R18 tire in the top plot of Figure 2.8. In both plots, we see that the effects due to the tread pattern cause asperities in the exact shape and vertical pressure distribution. However, these asperities are primarily an artifact of taking a single, static pressure measurement. In operation, the tire's rotation tends to "average out" the effects of the tread pattern. This is why all brush tire models neglect tread patterns and typically assume a smooth pressure distribution.

As the figures show, this has a more pronounced effect on the shape of the Metzeler tire than the Avon tire, distorting it from the nominal elliptical shape. In operation, this shape would likely "average out" and become more like an ellipse. However, when using only static measurements, this complicates contact patch analysis. Therefore, the Metzeler tire is not used for many of the contact patch measurements in this chapter.

Figure 2.12 illustrates contact patches for a Dunlop 180/55R17 tire loaded on a flat surface with varying camber angles at a nominal load of $F_z = 2000$ N. The different nominal load was chosen since the load rating of the Avon tire is about 50% higher. Similar to the Avon, the tread patterns of the tires cause some irregularities, but overall the shapes of both the Avon and Dunlop are well-represented by ellipses. Because the two tires are such different sizes, the aspect ratios of the ellipses are quite different - the Dunlop is more elongated than the Avon.

Nominal Load and Inflation Pressure, Varying Camber Angle

Figure 2.8 illustrates contact patches for an Avon 300/35R18 tire at its nominal load and inflation pressure on a flat surface for varying camber angles. Likewise, Figure 2.12 illustrates the contact patches at similar conditions for a Dunlop 180/55R17 tire.

The top plots of each figure show the two tires at zero camber, and were discussed above. The middle plots of each are at a camber angle of $\gamma = 20^{\circ}$. The biggest difference between the plots is the shape of the tread pattern. Because the tire is being loaded at an angle, a different part of the tread pattern is used in the contact patch. Otherwise, there is little difference between the $\gamma = 0^{\circ}$ and $\gamma = 20^{\circ}$ cases. The elliptical contact patch shape and size do not vary appreciably.

The bottom plots of each figure are at a camber angle of $\gamma = 40^{\circ}$. Here, the inboard side of the contact patch begins to flatten (the bottom edges of the plots). This is because the tire sidewall is beginning to enter the contact patch, which is more pronounced on the Avon tire than the Dunlop tire. This effect can be further exaggerated by increasing normal load and camber angle, resulting in the photograph in Figure 2.6. Here, it is clear that the tire sidewall dramatically changes the shape of the contact patch and reduces its area.

Operating at conditions that cause the tire sidewall to distort the contact patch tend to degrade the tire's performance. Therefore, if one wants maximum tire capability (e.g. high peak lateral force), this is not a desirable operating region for the tire.



Figure 2.6: A Dunlop motorcycle tire at $F_z = 3600$ N, P = 2.4 bar, and $\gamma = 45^{\circ}$, illustrating contact patch distortion due to the tire sidewall

This is reflected in the design of the Dunlop tire. Because it is designed specifically for high-performance sports motorcycles, it is able to withstand $\gamma = 40^{\circ}$ at typical operating conditions before starting to see the onset of sidewall distortion.

Because the sidewall carries much of the vertical load in these cases, they are not well-characterized by the contact patch models derived in Section 2.3. However, because this is not the desired operating region of a tire, this is not a significant model limitation.

Zero Camber Angle, Varying Load and Inflation Pressure

Figure 2.9 illustrates contact patches for an Avon 300/35R18 tire loaded on a flat surface with zero camber at different loading conditions. Compared to the nominal loading case used for the camber plots in Figure 2.8, one has a lighter normal load, one has a heavier normal load, and one has a higher inflation pressure. Similarly, Figure 2.13 provides the same three cases for a Dunlop 180/55R17 tire.

In each of these plots, the contact patch size changes relative to the nominal

loading case: increasing load increases the area, decreasing the load decreases the area, and increasing the inflation pressure decreases the area. Because of this, more or less of the tread pattern may be included in the contact patch. However, the overall elliptical shape and aspect ratio do not change significantly.

Loading on a Drum

The same tests as illustrated in Figures 2.8 and 2.9 for the Avon tire and Figures 2.12 and 2.13 for the Dunlop tire are repeated in Figures 2.10 and 2.11 for the Avon tire and Figures 2.14 and 2.15 for the Dunlop while loaded onto a ϕ 1.2 m drum, as opposed to a flat surface. This emulates the effect of the rolling road used for the suspension prototype (see Section 5.3.3). In all cases, the effect is a slight widening in the lateral direction and a large shortening in the longitudinal direction. The result is a decrease in both contact patch area and aspect ratio. Additionally, because the contact patch becomes wider, the effect of the sidewall at high camber angles is exaggerated. In fact, Figure 2.10 shows some evidence of sidewall distortion even at a camber angle of $\gamma = 20^{\circ}$.



Figure 2.7: Contact patch of a Metzeler 300/35R18 motorcycle tire



Figure 2.8: Contact patches of an Avon $300/35\mathrm{R}18$ motorcycle tire at different camber angles on a flat surface



Figure 2.9: Contact patches of an Avon 300/35R18 motorcycle tire at different normal loads and inflation pressures on a flat surface



Figure 2.10: Contact patches of an Avon 300/35R18 motorcycle tire at different camber angles on a $\phi 1.2$ m drum



Figure 2.11: Contact patches of an Avon 300/35R18 motorcycle tire at different normal loads and inflation pressures on a $\phi 1.2$ m drum


Figure 2.12: Contact patches of a Dunlop 180/55R17 motorcycle tire at different camber angles on a flat surface



Figure 2.13: Contact patches of a Dunlop 180/55R17 motorcycle tire at different normal loads and inflation pressures on a flat surface



Figure 2.14: Contact patches of a Dunlop 180/55R17 motorcycle tire at different camber angles on a $\phi 1.2$ m drum



Figure 2.15: Contact patches of a Dunlop 180/55R17 motorcycle tire at different normal loads and inflation pressures on a $\phi 1.2$ m drum

2.3 Contact Patch Model

This section begins by developing two physically-based contact patch models. These predict the contact patch size, shape, and vertical force distribution using only tire geometry, normal load F_z , and inflation pressure P. These two models share many of the same modeling assumptions, and yield the same prediction for size and shape, but differ in the assumptions used to determine the vertical pressure distribution. The first model, the linear deformation model, assumes that the pressure distribution is determined by the amount by which the tire ring/band deforms to form the contact patch. This results in a pressure distribution that is well-approximated by a paraboloid. The second model, the balloon model, assumes that the tire band is thin and has no impact on the resulting pressure distribution. The resulting pressure distribution is therefore uniform, being equal to the inflation air inside the balloon-like tire band.

From the measurements taken in Section 2.4, it is clear that the actual vertical pressure distributions are somewhere between the predictions of these two physically-based models. Therefore, a semi-empirical model is developed to blend these two physically-based models.

2.3.1 Contact Patch Definitions

The contact patch shape considered in both the physically-based and semi-empirical models is that of an ellipse, shown in Figure 2.16. The x and y axes are in the plane of the road, with the x-axis aligned with the forward direction of the tire and the y-axis pointing left. The z-axis is pointing upward. The half-length is a and the half-width is b.

The contact patch models developed in this section are symmetric about both the x- and y-axes. Although not considered in this thesis, the tire brush model developed in Chapter 3 could be applied more generally, allowing any arbitrary contact patch shape that is symmetric about the y-axis (but not necessarily the x-axis).



Figure 2.16: Diagram of an elliptical contact patch

2.3.2 Physically-Based Models

Contact Patch Size and Shape

By using tire geometry and loading conditions (inflation pressure P and normal load F_z), the contact patch size and shape can be predicted. This prediction is based off of the following assumptions:

- Assume the road is flat and rigid. This means that the contact area between the tire and the road will be a flat plane. Also, this implies that deformation occurs only in the tire.
- Assume that only the inflation air carries the vertical load. This means that the contact patch area A_{est} can be estimated using only the normal force F_z and the inflation pressure P as follows:

$$A_{est} = \frac{F_z}{P} \tag{2.1}$$

• Press the tire into the road until the correct contact area is obtained.

This deforms the tire vertically by a distance h, generating a flat contact patch at its bottom. The resulting vertical deformation profile is determined by tire geometry and is given by $\delta_z(x, y)$.

The results in Section 2.4.1 illustrate when the assumption about inflation pressure carrying the normal load is valid. In general, the *size* of the contact patch is wellpredicted by this assumption for most normal loading conditions. However, if the tire is severely overloaded, under-inflated, or highly-cambered, the sidewall begins to carry a notable portion of the normal load and the accuracy of this estimate is eroded. Also, if the tire is placed on a drum instead of a flat road, the estimate needs to be modified.

The last assumption determines the *shape* of the contact patch. For toroid tires, this shape is well-approximated by an ellipse calculated using the effective overall radius r_{te} and tread profile radius r_{tt} (See Figure 2.1). A derivation for this is given in Appendix A. The results are as follows:

$$1 - \left(\frac{x}{a}\right)^2 - \left(\frac{y}{b}\right)^2 = 0 \tag{2.2}$$

$$a = \sqrt{\frac{F_z}{\pi P}} \sqrt[4]{\frac{r_{te}}{r_{tt}}}$$
(2.3)

$$b = \sqrt{\frac{F_z}{\pi P}} \sqrt[4]{\frac{r_{tt}}{r_{te}}}$$
(2.4)

$$\frac{a}{b} = \sqrt{\frac{r_{te}}{r_{tt}}} \tag{2.5}$$

Vertical Pressure Distribution - Linear Deformation Model

The linear deformation model assumes that the vertical force distribution $\sigma_z(x, y)$ is proportional to the vertical deformation $\delta_z(x, y)$ needed to generate the contact patch. A flat plane, representing the road, is pressed into the tire until the resulting contact area matches the estimate from Equation 2.1. The vertical/normal pressure at any point is modeled as proportional to the amount of deformation required to attain this flat contact patch. For toroid tires, the vertical force distribution $\sigma_z(x, y)$ is wellapproximated by a paraboloid. The derivation in Appendix A yields the following results:

$$\sigma_z(x,y) = \frac{2F_z}{\pi ab} \left(1 - r^2\right) \tag{2.6}$$

$$r = \sqrt{\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2} \tag{2.7}$$

This model is illustrated in Figure 2.17. Note that for these plots, and all like them in the remainder of this thesis, the leading edge of the contact patch is on the right-hand side of the figure.



Figure 2.17: Vertical pressure distribution predicted by the linear deformation model

If the same assumptions are applied to a car tire (see Section 2.5), the results are similar to the 1D parabolic pressure distributions used in many 1D tire brush models (see Section 1.2.1)

Vertical Pressure Distribution - Balloon Model

The balloon model assumes that the tire band is thin and has no impact on the resulting pressure distribution. This means that the vertical pressure distribution matches that of the inflation air on the opposite side of the tire band, resulting in a uniform distribution:

$$\sigma_z(x,y) = \frac{F_z}{\pi ab} \tag{2.8}$$

If the same assumptions are applied to a car tire (see Section 2.5), the results are similar to the 1D uniform pressure distributions used in many 1D tire brush models (see Section 1.2.1).

2.3.3 Semi-Empirical Model

This thesis presents a semi-empirical model inspired by the two physically-based models above, but extended to permit a better representation of experimental data. It is developed using the physically-based models as a starting point, but accounting for three additional effects:

- Role of tire band compliance. Data illustrate that actual contact patches exhibit vertical pressure distributions that are somewhere between the predictions of the linear deformation model and the balloon model. Therefore, the semi-empirical model is parameterized to account for a continuum of band compliances from the linear deformation model to the balloon model.
- Longitudinal and lateral asymmetry. Data illustrate that the role of tire band deformation is different in longitudinal and lateral directions. This is largely the result of tire construction, which is often asymmetric. As a result, the longitudinal and lateral band compliances are parameterized separately in the semi-empirical model. For example, many tires have radial belts which tend to make the longitudinal direction stiffer than the lateral direction. This would result in a longitudinal band compliance closer to that predicted by the linear deformation model than the lateral band compliance.

• Variation in contact patch size and shape. Data illustrate that, for contact patches that are not significantly distorted by the sidewall, the contact patch shape is well-approximated by an ellipse. However, the exact sizes and shapes of these ellipses are not always accurately predicted by the physically-based models. Therefore, to be more widely applicable, the semi-empirical model leaves these as free parameters. This is similar to what is typically done for 1D brush tire models, where the size and shape of the (typically rectangular) contact patch are also left as parameters.

The resulting model has five parameters $(a, b, n, m, \text{ and } F_z)$, and is given as:

$$\sigma_z(x,y) = M\left(1 - r^n \cos^2 \theta\right) + M\left(1 - r^m \sin^2 \theta\right)$$
(2.9)

$$r = \sqrt{\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2} \tag{2.10}$$

$$\theta = \tan^{-1} \frac{y/b}{x/a} \tag{2.11}$$

$$M = \frac{F_z}{\pi ab} \left(1 - \frac{1}{n+2} - \frac{1}{m+2} \right)^{-1}$$
(2.12)

where a and b are the longitudinal and lateral half-lengths of the contact patch ellipse (see Figure 2.16), and n and m are the longitudinal and lateral tire band compliance parameters. The vertical pressure $\sigma_z(x, y)$ is restricted to be non-negative. The elliptical shape of the contact patch is found by setting $\sigma_z(x, y) = 0$, and allowing a and b to deviate from the physically-based predictions permits variation in contact patch size and shape.

One way to analyze the effects of different n and m is by plotting the longitudinal $(p_x(x))$ and lateral $(p_z(y))$ normalized pressure distributions. This is particularly useful when fitting models to data since it deemphasizes irregularities caused by tread elements (see Section 2.4.2). These distributions are found by integrating along the x and y directions. Additionally, x is normalized as x/a, y as y/a, and $\sigma_z(x, y)$ as

 $\sigma_z(x,y)/F_z$. The resulting normalized pressure distributions are given as:

$$p_x(x/a) = a \int \frac{\sigma_z(x,y)}{F_z} dy \qquad (2.13)$$

$$p_y(y/b) = b \int \frac{\sigma_z(x,y)}{F_z} dx \qquad (2.14)$$

The result of normalization is that each of these is defined on [-1, 1] and $\int p_x(x/a) dx = \int p_y(y/b) dy = 1.$

Figure 2.18 illustrates the effects of varying n and m symmetrically. The linear deformation model is expressed using n = m = 2 and the balloon model is expressed using $n = m = \infty$. Varying n and m symmetrically between these values allows the continuum of tire band compliance to be expressed. This is similar to the effect used in some 1D brush models, where the model of the vertical pressure distribution can be altered by varying the shape of a trapezoid or exponential (see Section 1.2.1).



Figure 2.18: Pressure distributions resulting from varying n and m symmetrically

Setting $n \neq m$ accounts for longitudinal and lateral asymmetry, as is illustrated in Figure 2.19. Here, n = 2 is fixed and m is varied. This illustrates that the normalized longitudinal pressure distribution p_x is largely independent of m. The opposite is also true: the normalized lateral pressure distribution is largely independent of n.



Figure 2.19: Pressure distributions resulting from varying n and m asymmetrically

2.4 Contact Patch Analysis

2.4.1 Analysis of Size and Shape

Most brush models do not typically predict the exact contact patch size and shape, instead leaving them as free parameters. However, the data in Section 2.2 showed significant trends that illustrate how the contact patch size and shape (given by parameters a and b) may be expected to change for operating conditions other than those specifically measured. These can be analyzed by comparing them against the predictions of the physically-based contact patch models, helping inform the parameter choices one should use in the semi-empirical model.

Tables 2.2 and 2.4 show how the contact patch *size* and Tables 2.3 and 2.5 show how the contact patch *shape* change with loading conditions for the Dunlop and Avon tire, respectively. These values are compared against those predicted by the assumptions of the physically-based models in Section 2.3.2. Note that these measurements do not necessarily assume an elliptic shape; the area A is measured directly, and the contact patch half-lengths a and b are found by measuring the overall length and width.

For the Dunlop tire on flat surface, the contact patch area is close to the prediction A_{est} from Equation 2.1 for varying loads F_z and inflation pressures P. This is true for both small and large camber angles. The shape and aspect ratio of the Dunlop contact patch are also similar to their model predictions. This indicates that the effect of the sidewall on the contact patch is small, consistent with Figure 2.12.

The contact patch of the Avon tire is not as easily predicted as the Dunlop tire. On a flat surface, it appears to vary consistently with expectation for varying normal load F_z , but not necessarily with inflation pressure P. Additionally, at high camber angles the area decreases notably. This is because the sidewall has entered the contact patch and is carrying a significant part of the load, consistent with Figure 2.8. The aspect ratio of the Avon contact patch, although consistent across varying F_z and P, is not close to the model prediction.

The contact patches of both tires, when loaded onto a drum, grow slightly in the lateral direction (b) and shrink significantly in the longitudinal direction (a), causing an overall decrease in area (A) by about 20%. The resulting aspect ratios appear to be consistent, but notably lower than the model prediction.

γ	road	F_z	P	A	A_{est}	A_{est} error
(°)	surface	(N)	bar	(cm^2)	(cm^2)	(%)
0	flat	900	2.4	39	38	-2.6
0	flat	2000	2.4	86	83	-3.5
0	flat	3000	2.4	129	125	-3.1
0	flat	2000	2.9	71	69	-2.8
20	flat	2000	2.4	86	83	-3.5
40	flat	2000	2.4	85	83	-2.3
0	drum	1000	2.4	39	46	18.0
0	drum	2000	2.4	68	83	22.0
0	drum	3300	2.4	114	138	20.6
0	drum	2000	2.9	59	69	17.0
20	drum	2000	2.4	73	83	13.7
40	drum	2000	2.4	71	83	16.9

Table 2.2: Measurements of contact patch size for a Dunlop $180/55\mathrm{R}17$ motorcycle tire

road	F_z	P	a	b	$\frac{a}{b}$	a_{est}	b_{est}	$\frac{a_{est}}{b_{est}}$
surface	(N)	(bar)	(mm)	(mm)		(mm)	(mm)	
flat	900	2.4	47	26	1.83	45	26	1.73
flat	2000	2.4	70	40	1.74	68	39	1.73
flat	3000	2.4	84	50	1.70	83	48	1.73
flat	2000	2.9	63	37	1.71	68	39	1.73
drum	1000	2.4	39	29	1.34	48	28	1.73
drum	2000	2.4	51	43	1.17	68	39	1.73
drum	3300	2.4	66	56	1.18	87	50	1.73
drum	2000	2.9	48	39	1.22	68	39	1.73

Table 2.3: Contact patch dimensions for a Dunlop 180/55R17 motorcycle tire at $\gamma=0$ camber angle

γ	road	F_z	P	A	A_{est}	A_{est} error
(°)	surface	(N)	bar	(cm^2)	(cm^2)	(%)
0	flat	2000	2.4	89	83	-6.7
0	flat	3000	2.4	132	125	-5.3
0	flat	4000	2.4	178	167	-6.2
0	flat	3100	2.9	120	107	-10.8
20	flat	3000	2.4	136	125	-8.1
40	flat	3000	2.4	105	125	19.1
0	drum	2000	2.4	74	83	12.2
0	drum	3000	2.4	108	125	15.7
0	drum	4000	2.4	145	167	15.2
0	drum	3100	2.9	106	107	0.9
20	drum	3000	2.4	113	125	10.6
40	drum	3000	2.4	89	125	40.4

Table 2.4: Measurements of contact patch size for an Avon $300/35\mathrm{R}18$ motorcycle tire

road	F_z	P	a	b	$\frac{a}{b}$	a_{est}	b_{est}	$\frac{a_{est}}{b_{est}}$
surface	(N)	(bar)	(mm)	(mm)		(mm)	(mm)	
flat	2000	2.4	53	52	1.01	58	46	1.27
flat	3000	2.4	66	64	1.03	71	56	1.27
flat	4000	2.4	77	76	1.01	82	65	1.27
flat	3100	2.9	60	63	0.96	72	57	1.27
drum	2000	2.4	45	57	0.79	58	46	1.27
drum	3000	2.4	53	71	0.74	71	56	1.27
drum	4000	2.4	61	82	0.75	82	65	1.27
drum	3100	2.9	55	68	0.81	72	57	1.27

Table 2.5: Contact patch dimensions for an Avon 300/35R18 motorcycle tire at $\gamma=0$ camber angle

2.4.2 Analysis of Vertical Pressure Distribution

Fitting the vertical pressure distribution of contact patches to the semi-empirical model is difficult to do directly. This is because the tread patterns cause significant irregularities in contact pressure measurement. One way to deemphasize the tread pattern is to symmetrize the measurements about both the x- and y- axes. After significant downsampling, the resulting 3D plots for the Dunlop and Avon tires on a flat surface at their nominal loading conditions are pictured in Figures 2.20 and 2.21, respectively. However, determining the correct tire band compliance parameters (n, m) by comparing these to similar plots from contact patch models, such as the one in Figure 2.17, is still difficult.



Figure 2.20: Vertical pressure distribution for a Dunlop 180/55R17 tire after symmetrizing and downsampling

As discussed in Section 2.3.3, a more clear way to analyze the vertical pressure



Figure 2.21: Vertical pressure distribution for an Avon 300/35R18 tire after symmetrizing and downsampling

distribution is by fitting the longitudinal and lateral normalized pressure distributions (p_x, p_y) , given by Equations 2.13 and 2.14, respectively. By symmetrizing the contact patch measurements and plotting p_x and p_y , it is notably more clear which are the correct semi-empirical model parameters for the contact patches.

Figure 2.22 illustrates the results for a Dunlop 180/55R17 tire at zero camber on both a flat surface (top) and a ϕ 1.2 m drum (bottom). Figure 2.23 illustrates the same results for an Avon 300/35R18 tire. Each plot has four sets of data, corresponding to the nominal loading case, the lighter load, the heavier load, and the higher inflation pressure. Figure 2.24 illustrates the Metzeler 300/35R18 tire, but only at the nominal loading case (since the Metzeler contact patch was not tested at all conditions). Figures 2.20 and 2.21, the irregularities caused by tread pattern are notably deemphasized. Because of the tread design, this is more true for the longitudinal distributions than the lateral distributions. For example, there are no Dunlop tire treads along the centerline y = 0 whereas the large tread groove of the Avon tire does cross the centerline y = 0. As a result, the lateral distribution p_y at y = 0 shows a slight hump for the Dunlop tire and a slight valley for the Avon tire.

These plots illustrate that the normalized longitudinal and lateral pressure distributions do not change significantly with normal load F_z and inflation pressure P. This means that their effects are captured by varying the contact patch shape and size (a, b) while holding the tire band compliance factors constant (n, m).

This is not the case for loading the tire onto a flat surface versus a drum. Loading the tire on a drum causes a notable change to the longitudinal pressure distribution p_x , but not to the lateral pressure distribution p_y . This reflects the shape of the drum itself, which has a very different curvature from a flat surface in the longitudinal direction but not the lateral direction. This is captured in the model by a decrease in n and no change in m.

The resulting n and m parameters are summarized in Table 2.6. It's interesting to note that the three tires all have different parameters. The tire that has the lowest lateral band compliance parameter m is the Dunlop tire, which is designed for high-performance sports motorcycles. This indicates that its lateral distribution is closest to parabolic of the group, which has implications on lateral force generation capabilities (see Chapter 3).

The Avon and Metzeler tires, which are of near identical size, have different n and m parameters. This is consistent with their differing constructions, as discussed in Section 2.1. The Metzeler tire carcass is designed with radial steel belts, which have the effect of increasing longitudinal band stiffness much more than lateral bandd stiffness. By contrast, the Avon tire carcass lacks radial steel belts and instead uses many more non-steel bias layers that increase both longitudinal and lateral stiffness. (As an aside, these results are consistent with the experience of mounting the tires to their rims: the higher stiffness of the Avon sidewalls made it much more difficult to mount!)



Figure 2.22: Longitudinal and lateral vertical pressure distributions for a Dunlop 180/55R17 motorcycle tire at $\gamma = 0^{\circ}$ on a flat surface (top) and a ϕ 1.2 m drum (bottom)



Figure 2.23: Longitudinal and lateral vertical pressure distributions for an Avon 300/35R18 motorcycle tire at $\gamma = 0^{\circ}$ on a flat surface (top) and a $\phi 1.2$ m drum (bottom)



Figure 2.24: Longitudinal and lateral vertical pressure distributions for a Metzeler 300/35R18 motorcycle tire at $\gamma = 0^{\circ}$ on a flat surface

tire	tire	road		
brand	size	surface	n	m
Dunlop	180/55R17	flat	6	4
Dunlop	180/55R17	drum	4	4
Avon	300/35R18	flat	4	8
Avon	300/35R18	drum	2	8
Metzeler	300/35R18	flat	2	10

Table 2.6: Semi-empirical model parameters for tire band compliance

2.5 Comparison to Car Tires

The geometry of a car tire is very different than that of a motorcycle tire, being more similar to a cylinder (of finite length, representing the contact patch width) than a toroid. Although not useful for active camber concepts, its useful to consider how the same modeling principles apply to these tires.

This section extends the semi-empirical model to cylindrical car tires, which can then be used for developing 1D tire brush models [39] [12] [14]. Data is presented from a racing slick, which is intended for competition use and is nearly treadless. This significantly reduces the irregularities caused by treads, simplifying analysis. Finally, the normalized longitudinal and lateral pressure distributions are used to analyze the data and perform a model fit.

Unlike motorcycle tires, there are many other contact patch measurements of cylindrical tires in existing literature. For example, Lippmann [30] and Sakai [46] provide data for radial passenger car tires and Kvatinsky et al [26] provide data for a large aircraft tire. As was the case with motorcycle tires, the vertical pressure distributions on flat surfaces look neither parabolic nor uniform; it appears that a better characterization would be found by applying the semi-empirical model since it allows a band compliance parameter between these two extremes.

2.5.1 1D Contact Patch Model

The semi-empirical model can be extended to car tires by making two changes:

- Rectangular contact patch shape. Instead of assuming an elliptical contact patch as with motorcycle tires, the assumed contact patch shape for car tires is a rectangle. This is essentially the same shape used for the basis of many 1D brush tire models.
- Uniform lateral distribution. For purposes of developing brush models for car tires, dependence of the vertical pressure distribution on y is often ignored. This results in a 1D tire models where only variation in x is considered.

The resulting model has three parameters (a, b, and n), and is given as:

$$\sigma_z(x,y) = M\left(1 - \left|\frac{x}{a}\right|^n\right)$$
(2.15)

$$M = \frac{F_z}{4ab} \left(1 - \frac{1}{n+1} \right)^{-1}$$
(2.16)

where a and b are the longitudinal and lateral half-lengths of the contact patch rectangle, and n is the longitudinal tire band compliance parameter. The vertical pressure $\sigma_z(x, y)$ is defined on $x \in [-a, a]$ and $y \in [-b, b]$.

If n = 2, similar to the linear deformation model, then the model predicts a parabolic pressure distribution which is used in many brush models such as those of Fiala [14] and Pacejka [39]. If $n = \infty$, similar to the balloon model, then the model predicts a uniform distribution which is also used in many brush models, such as that of Dugoff [12].

2.5.2 Contact Patch Measurements and Analysis

Figure 2.26 presents the contact patches for the Hoosier 225/45R17 racing slick at camber angle $\gamma = 0^{\circ}$. It is shown at a nominal normal load on a flat surface, a heavy normal load on a flat surface, and a nominal normal load on a $\phi 1.2$ m drum. As can be seen in the figure, the tire is completely slick and treadless with the exception of two circumferential grooves. Similar to the motorcycle tires, the contact patch grows when the normal load is increased and shrinks when loaded on a drum.

What's significantly different from a motorcycle tire is the response of the car tire to cambering. Figure 2.25 shows the Hoosier 225/45R17 racing slick with a small camber angle of $\gamma = 3^{\circ}$. Even at this small camber angle, the contact patch shape is altered significantly. Certainly, these types of tires are not suitable for use at camber angles much over $5^{\circ} - 8^{\circ}$ or so, making them unsuited for use on an active camber concept.

Table 2.7 provides a comparison of the actual contact patch area A and the estimated area A_{est} from Equation 2.1. The results are similar to that of the motorcycle tires: on flat road, the estimate A_{est} is quite close to the measured area A, but decreases by about 20% when placed on a drum.

To analyze the semi-empirical model fit, the longitudinal and lateral normalized pressure distributions (p_x, p_y) are symmetrized and plotted in Figure 2.27. There are two conditions plotted for the flat surface (nominal and heavy loads) and one condition for the drum (nominal load). Since the model prediction is a rectangular contact patch and a uniform lateral distribution, the predicted lateral normalized pressure distribution p_y is completely flat. This is mostly valid, with the exception of the regions near the two circumferential grooves and the edges of the contact patch. As with motorcycle tires, the effect of the drum is to decrease n with little/no impact on the lateral pressure distribution.

γ	road	F_z	P	A	A_{est}	A_{est} error
(°)	surface	(N)	bar	(cm^2)	(cm^2)	(%)
0	flat	3600	2.4	155	148	-4.5
0	flat	5400	2.4	226	221	-2.2
3	flat	3600	2.4	153	148	-3.3
0	drum	3600	2.4	126	148	17.5

Table 2.7: Measurements of contact patch size for a Hoosier 225/45R17 racing car tire



Figure 2.25: Contact patch of a Hoosier 225/45R17 racing car tire at $\gamma = 3^{\circ}$ camber angle



Figure 2.26: Contact patches of a Hoosier 225/45R17 racing car tire at different normal loads and surfaces



Figure 2.27: Longitudinal and lateral vertical pressure distributions for a Hoosier 225/45R17 racing car tire at $\gamma = 0^{\circ}$ on a flat surface (top) and a $\phi 1.2$ m drum (bottom)

Chapter 3

Brush Tire Model

Contact patch friction is optimized when the lateral force distribution and vertical force distribution match. This means that the tire is making use of all available friction in the contact patch for generating lateral force. Although not an exact match, the lateral force distribution from camber is a much closer match than is the one from slip angle. Because the ultimate goal of the active camber concept is to generate lateral forces almost exclusively from camber, this could provide the vehicle with 20 - 30% additional lateral force capability and decreased transient response time using specialized tires.

To attain high camber angles, the tires will have a rounded profile, similar to a motorcycle tire. Contact patches for passenger car tires are approximately rectangular. Therefore, most tire models treat the contact patch only in 1D, assuming uniformity in the lateral direction. Contact patches for motorcycle tires are approximately elliptic. This cannot be adequately addressed by 1D tire models.

Therefore, this chapter develops a new variant of a brush tire model that considers a 2D contact patch. This model is based on many of the same principles used in 1D brush tire models. However, a 2D contact patch representation is required to adequately model force generation from camber. This is because camber induces not only lateral but longitudinal deformation in the contact patch, which cannot be captured in a 1D brush tire model.

The model is compared to experimental data taken with the three motorcycle tires

described in Section 2.1. The results show that the model does capture the effects of camber and slip angle on motorcycle tires. The parameterization predicted by the model indicates that, as expected (see Section 1.2.1), motorcycle tires do not exhibit significant increases in lateral force by using camber as opposed to slip angle.

Finally, the tire model is used to hypothesize the design of a specialized tire for the active camber concept in Section 3.6 that *does* exhibit significant increase in lateral force by using camber as opposed to slip angle. These specially-designed tires would be optimized to support the high loads of such a vehicle and maximize friction in the contact patch. Furthermore, these tires would attain this peak friction with relatively little camber. Typical motorcycle tires require upwards of 45° camber to attain their peak lateral force, whereas specialized tires should require about half of that camber amount, significantly simplifying suspension design and packaging. Similar to the F400 (see Section 1.1.3), the model predicts that this specialized tire should provide nearly 30% more lateral force by using camber instead of slip angle.

3.1 Overview

The basic idea of the model is to explain steady-state tire force generation from a set of slip conditions using several small, independent brushes. The schematic of the model is given in Figure 1.19. The ring or band of the tire is assumed rigid in the lateral and longitudinal directions. The carcass, which connects the rigid band to the rigid wheel, is modeled as having a linear longitudinal stiffness K_{cx} , lateral stiffness K_{cy} , and torsional stiffness $K_{c\alpha}$. The bases of the brushes are attached to the rigid band. The tips of the brushes touch the road and deform due to slip velocities. The brushes are assumed to be of negligible height.

By applying brush stiffness and friction models, these brush deformations can be translated to force distributions. Finally, these force distributions can be integrated to give tire curves.

As discussed in Section 1.2.1, typically brush models in the literature are only 1D. They consider a single row of longitudinal brushes, similar to the illustration in Figure 1.18. These models implicitly assume a rectangular contact patch with uniform pressure distribution in the lateral direction. This can provide a good characterization of passenger car tires.

However, it is not sufficient for rounded tires similar to motorcycle tires. This is because the contact patches of these tires are not well-described by a 1D model variation in both longitudinal and lateral directions must be considered. Furthermore, a 1D model cannot thoroughly represent deformation from camber. Therefore, this chapter develops a 2D brush model, which does not have these limitations.

This brush model can be broken down into the following steps:

- 1. Determine contact patch shape and vertical pressure distribution. This was developed in Chapter 2.
- 2. Determine ideal brush deformation. This is calculated from the tire slip conditions and is developed in Section 3.3.
- 3. Determine tire forces. This is calculated by applying stiffness and friction models to the deformations to generate force distributions, then integrating over these distributions to get tire forces and moments. This is developed in Section 3.4.
- 4. Determine tire carcass deformation. This is calculated using the resulting tire forces and moments above. This is discussed in Section 3.2. Because tire carcass deformation changes the ideal brush deformation, iteration is used to find the solution.

Note that this sequence of steps is similar for most variants of both 1D and 2D brush models (see Section 1.2.1).

3.1.1 Definitions and Assumptions

As discussed in Chapter 2, the basic tire shape considered in this paper is a toroid, resulting in an elliptical contact patch shape. A schematic of the toroid is given in Figure 2.1. The tire effective major radius is r_{te} and the tread profile radius is r_{tt} .

The coordinate system and tire velocities are illustrated in Figure 3.1. The x and y axes are in the plane of the road, with the x-axis aligned with the forward direction of the tire, the y-axis pointing left, and the origin at the center of the (undeformed) contact patch on the ground. The z-axis is pointing upward. The wheel has a camber angle rotation of γ about the x-axis.



Figure 3.1: Diagram of coordinate system and tire velocities

The wheel velocities are given as:

- V_x : Longitudinal velocity. This is measured along the forward x-axis.
- V_y : Lateral velocity. This is measured along the leftward y-axis.
- r: Yaw rate. This is the rotational speed of the coordinate frame relative to the road measured along the upward z-axis. When $\gamma = 0$, this is also the rotational speed of the wheel itself.

• Ω : Wheel rotational speed. This is the rotational speed of the tire, measured about the wheel's axis. When $\gamma = 0$, this is aligned with the *y*-axis. If $\gamma \neq 0$, then some component of this rotation is along the *z*-axis.

The absolute rotational velocities of the wheel are given as:

$$\omega_u = \Omega \cos \gamma \tag{3.1}$$

$$\omega_z = \Omega \sin \gamma + r \tag{3.2}$$

For the steady-state brush model, it is assumed that the vertical velocity V_z and camber rate ω_x are zero.

The total resulting tire forces and moments acting on the tire from the road are:

- F_x : longitudinal tire force, measured along the forward x-axis.
- F_y : lateral tire force, measured along the leftward y-axis.
- F_z : vertical tire force, measured along the upward z-axis.
- M_x : overturning moment, measured along the forward x-axis.
- M_y : drive moment, measured along the wheel's axis.
- M_z : aligning moment, measured along the upward z-axis.

Note that tire rolling resistance, often small in practice, is neglected in this model.

A schematic of the contact patch with carcass and brush deformations is given in Figure 3.2. Tire carcass deformation results in bulk movement of the contact patch by a distance ε_x longitudinally, a distance ε_y laterally, and and angle ε_α torsionally. In the figure, a brush element located at (x_b, y_b) is deformed longitudinally by δ_x and laterally by δ_y .

Information about the leading and trailing edge are encoded into the traveled distance function $s(x_b, y_b)$ and the contact patch half-length function $d(y_b)$. The maximum half-length is *a* and the maximum half-width is *b*. These are illustrated for an elliptical contact patch in Figure 3.3. Note that the functions $s(x_b, y_b)$ and $d(y_b)$



Figure 3.2: Schematic of brush and carcass deformation

are related as follows:

$$d(y_b) = s(x_b, y_b) + x_b (3.3)$$

For an elliptical contact patch, these are given by:

$$d(y_b) = a\sqrt{1 - \left(\frac{y_b}{b}\right)^2} \tag{3.4}$$

$$s(x_b, y_b) = a \sqrt{1 - \left(\frac{y_b}{b}\right)^2 - x_b}$$
 (3.5)

The effective radius of a toroidal tire at a given camber angle γ is given as:

$$r_e(\gamma, y_b) = r_{te} - r_{tt} + \sqrt{r_{tt}^2 - (y_b \cos \gamma - r_{tt} \sin \gamma)^2}$$
(3.6)

where the tire effective major radius r_{te} and tread profile radius r_{tt} are as illustrated in Figure 2.1.



Figure 3.3: Diagram of an elliptical contact patch

3.2 Simplified Tire Carcass Model

As shown in Figure 3.2, the compliant tire carcass results in an offset of the contact patch from its nominal location. These offsets are given by:

$$\varepsilon_x = K_{cx}^{-1} F_x \tag{3.7}$$

$$\varepsilon_y = K_{cy}^{-1} F_y \tag{3.8}$$

$$\varepsilon_{\alpha} = K_{c\alpha}^{-1} M_z \tag{3.9}$$

where K_{cx} is the longitudinal carcass stiffness with corresponding offset ε_x , K_{cy} is the lateral carcass stiffness with corresponding offset ε_y , and $K_{c\alpha}$ is the torsional carcass stiffness with corresponding angular offset ε_{α} .

Because the magnitude of ε_{α} is small, typically no more than 2°, and usually $|V_y| \ll |V_x|$, the mapping of tire velocities V_x , V_y , and Ω in x, y coordinates to V_{xb} , V_{yb} , and Ω_b in x_b, y_b coordinates can be approximated using a small angle approximation. Note that because the model is considered only in steady state, there is no change to

yaw rate r.

3.3 Ideal Brush Deformation

The concepts behind brush deformation for the 2D brush model are the same as those for the 1D model. The primary difference is that, for the 1D model, only a single row of longitudinal brushes are considered.

For this part of the model, all friction limits are neglected. When in the contact patch, the tips of brushes maintain adhesion with the road.

The tire brushes are initially undeformed when they enter the contact patch at the leading edge. Slip velocities, due to mismatches between the tire carcass and road velocities, cause the brushes to deform as they move through the contact patch. At the trailing edge of the contact patch, the brush deformation returns to zero.

There are four different slip components to consider in the brush model:

- Longitudinal slip (κ). This is due to a mismatch between wheel rotational and translational speeds and causes longitudinal tire forces. This is typically actuated with powertrain or brake torque.
- Lateral slip angle (α). This is due to lateral movement of the wheel along the road and causes lateral tire forces. This is typically actuated by steering.
- Spin slip $(r/\sqrt{V_x^2 + V_y^2})$. This is due to the wheel spinning relative to the road and causes lateral tire forces. The magnitude of spin slip for typical steadystate maneuvers is very small, so it is often neglected in brush models. It is not discussed in the remainder of this chapter.
- Camber slip (γ) . This is a result of having a non-zero camber angle, which generates lateral tire forces.

Each of these components can be considered separately then added together for the complete brush deformation.

3.3.1 Longitudinal Slip

Longitudinal slip κ arises due to mismatch between wheel rotational and translational speeds, and is given by:

$$\kappa = \frac{r_e(\gamma, 0)\Omega - V_x}{V_x} \tag{3.10}$$

During free rolling, in the absence of other types of deformation, there is no longitudinal slip. During acceleration, the wheel spins faster, causing positive slip. During braking, the opposite occurs: the wheel spins slower, causing negative slip.

Using the approximations outlined in Section 3.2, Equation 3.10 can be expressed in x_b, y_b coordinates as:

$$\kappa = \frac{r_e(\gamma, 0)\Omega_b - V_{xb}}{V_{xb}} \tag{3.11}$$

The amount of longitudinal brush deformation δ_x per unit time is the slip velocity and is given by:

$$\frac{d\delta_x}{dt} = r_e(\gamma, 0)\Omega_b - V_{xb} \tag{3.12}$$

The length of contact patch travel x_b per unit time is the same as the tire carcass travel per unit time:

$$\frac{dx_b}{dt} = r_e(\gamma, 0)\Omega_b \tag{3.13}$$

Dividing these two equations provides the rate of deformation per unit contact patch length:

$$\frac{d\delta_x}{dx_b} = \frac{r_e(\gamma, 0)\Omega_b - V_{xb}}{r_e(\gamma, 0)\Omega_b} = \frac{\kappa}{1+\kappa}$$
(3.14)

This rate is multiplied by the traveled distance function $s(x_b, y_b)$ to get the resulting brush deformation as a function of contact patch position:

$$\delta_x(x_b, y_b) = s(x_b, y_b) \frac{\kappa}{1+\kappa}$$
(3.15)

Figure 3.4 shows a plot of the resulting brush deformation for a toroidal tire geometry. Note that this is different from what a 1D brush model would predict. This is because the 1D brush model inherently assumes a straight line for the leading edge. Therefore, $s(x_b, y_b) = a - x_b$, making brush deformation independent of lateral



position y_b .

Figure 3.4: Plot of longitudinal brush deformation δ_x due to longitudinal slip κ for the contact patch of a toroidal tire

3.3.2 Lateral Slip Angle

Lateral slip angle α arises due to lateral movement of the wheel along the road, and is given by:

$$\alpha = \arctan \frac{V_y}{V_x} \tag{3.16}$$

During free rolling, there is no slip angle. When the tire is cornering left, the tire moves to the right, generating negative slip angle. When the tire is cornering right, the opposite occurs: the tire moves to the left, generating positive slip angle.

Equation 3.10 can be expressed in x_b, y_b coordinates (without approximation) as:

$$\alpha_b = \arctan \frac{V_{yb}}{V_{xb}} = \alpha - \varepsilon_\alpha \tag{3.17}$$

The amount of lateral brush deformation δ_y per unit time is the slip velocity and
is given by:

$$\frac{d\delta_y}{dt} = -V_{yb} \tag{3.18}$$

The length of contact patch travel x_b per unit time is the same as the tire carcass travel per unit time:

$$\frac{dx_b}{dt} = r_e(\gamma, 0)\Omega_b \tag{3.19}$$

Dividing these two equations provides the rate of deformation per unit contact patch length:

$$\frac{d\delta_y}{dx_b} = \frac{-V_{yb}}{r_e(\gamma, 0)\Omega_b} = \frac{\tan \alpha_b}{1+\kappa}$$
(3.20)

This rate is multiplied by the traveled distance function $s(x_b, y_b)$ to get the resulting brush deformation as a function of contact patch position:

$$\delta_y(x_b, y_b) = s(x_b, y_b) \frac{\tan \alpha_b}{1+\kappa} = s(x_b, y_b) \frac{\tan (\alpha - \varepsilon_\alpha)}{1+\kappa}$$
(3.21)

Figure 3.5 shows a plot of the resulting brush deformation for a toroidal tire geometry. This is the exact same shape as is generated by longitudinal slip, except that the brush deformation is for the lateral direction, not the longitudinal one. Also similar to longitudinal slip, this is different from what a 1D brush model would predict. The straight leading edge of the 1D model would imply $s(x_b, y_b) = a - x_b$, making brush deformation independent of lateral position y_b .

3.3.3 Camber Slip

Camber slip arises due to the wheel traveling forward at a camber angle γ . During free rolling, there is no camber angle. A negative camber angle is used to corner left, and a positive camber is used to corner right.

A non-zero camber angle causes the tire carcass to travel in arcs over the road. This means that the tops of the brushes will travel in arcs, but the bottoms will travel in straight lines (since they are attached to the flat road). This is illustrated in Figure 3.6.

This induces brush deformations in both the lateral and longitudinal directions.



Figure 3.5: Plot of lateral brush deformation δ_y due to lateral slip angle α for the contact patch of a toroidal tire

The lateral deformation is because the arc pushes the tire carcass laterally in a curved path, with the peak deformation in the center of the contact patch. The longitudinal deformation is because the inside of the contact patch is moving slower than the outside.

The lateral deformation is derived by considering the geometry of a circle, inclined at the camber angle, projected downward into the plane of the road. From this, one can calculate the lateral position of a point on the contact patch (x_b, y_b) relative to the wheel center. This lateral position can also be calculated for the leading edge, which is when $x_b = d(y_b)$. Since the brush deformation is zero at the leading edge, subtracting these two lateral position measurements gives the resulting lateral brush deformation in the contact patch:

$$\delta_y(x_b, y_b) = -\sin\gamma \left[\sqrt{r_e(\gamma, y_b)^2 - x_b^2} - \sqrt{r_e(\gamma, y_b)^2 - d(y_b)^2} \right]$$
(3.22)

The longitudinal deformation is derived by considering the longitudinal velocity



Figure 3.6: Diagram of tire carcass paths through a cambered contact patch. The dotted lines are the straight lines of the road; the solid lines are the arcs taken by the tire carcass.

of a point (x_b, y_b) in the contact patch relative to the point on the centerline $(x_b, 0)$. The amount of longitudinal brush deformation δ_x per unit time is the slip velocity and is given by:

$$\frac{d\delta_x}{dt} = r_e(\gamma, y_b)\Omega_b - r_e(\gamma, 0)\Omega_b \tag{3.23}$$

The length of contact patch travel x_b per unit time is the same as the tire carcass travel per unit time:

$$\frac{dx_b}{dt} = r_e(\gamma, 0)\Omega_b \tag{3.24}$$

Dividing these two equations provides the rate of deformation per unit contact patch length:

$$\frac{d\delta_x}{dx_b} = \frac{r_e(\gamma, y_b)\Omega_b - r_e(\gamma, 0)\Omega_b}{r_e(\gamma, 0)\Omega_b} = \frac{r_e(\gamma, y_b)}{r_e(\gamma, 0)} - 1$$
(3.25)

This rate is multiplied by the traveled distance function $s(x_b, y_b)$ to get the resulting brush deformation as a function of contact patch position:

$$\delta_x(x_b, y_b) = s(x_b, y_b) \left(\frac{r_e(\gamma, y_b)}{r_e(\gamma, 0)} - 1\right)$$
(3.26)

This longitudinal deformation component is not captured by a 1D brush model with a single row of brushes. There are some ad hoc modifications to a 1D brush model that attempt to account for it (see Section 1.2.1), but none treat it directly. This is one of the primary advantages to using the 2D brush model.

Plots of the lateral and longitudinal deformation due to camber are shown in Figures 3.7 and 3.8 for the toroidal tire. Note that the color scales are slightly different for each figure. This is because while the lateral deformation is all positive, the longitudinal deformation is positive on one half of the tire and negative on the other half. This does not generate much longitudinal force, but it does use up tire friction that would have otherwise been available to generate lateral tire forces.



Figure 3.7: Plot of lateral brush deformation δ_y due to camber angle γ for the contact patch of a toroidal tire

However, in general, there is some small amount of net longitudinal force generated by this deformation. In fact, this is true even when $\gamma = 0$. This is entirely due to effective radius $r_e(\gamma, y_b)$ variation. However, in the absence of braking or drive torque, the net longitudinal force should be zero. To compensate for this, there should be a small, non-zero longitudinal slip ($\kappa \neq 0$), even in the absence of braking or drive



Figure 3.8: Plot of longitudinal brush deformation δ_x due to camber angle γ for the contact patch of a toroidal tire

torques. For typical tire geometries, the magnitudes of these numbers are all small.

Another important thing to consider is the effect of tread profile curvature r_{tt} on longitudinal deformation. A larger curvature will have less effective radius variation r_e over a given contact patch size, which will *decrease* longitudinal deformation. However, a larger curvature will also increase the width of the contact patch b, which will *increase* longitudinal deformation. In the end, this second effect dominates: overall, a larger tread profile curvature r_{tt} tends to *increase* the amount of longitudinal deformation in the contact patch.

3.4 Tire Forces

Once the ideal brush deformations δ_x and δ_y have been determined, they can be translated into tire force distributions. In general, one could apply any stiffness and friction model to this deformation profile.

The stiffness model used in this paper is a simple, constant stiffness model. This is consistent with the overwhelming majority of brush models in the literature. This gives the ideal longitudinal (σ_{xa}), lateral (σ_{ya}), and composite (σ_a) horizontal stresses. For these ideal horizontal stresses, complete adhesion is still assumed. They are given by:

$$\sigma_{xa}(x_b, y_b) = k_x \delta_x(x_b, y_b) \tag{3.27}$$

$$\sigma_{ya}(x_b, y_b) = k_y \delta_y(x_b, y_b) \tag{3.28}$$

$$\sigma_a(x_b, y_b) = \sqrt{\sigma_{xa}(x_b, y_b)^2 + \sigma_{ya}(x_b, y_b)^2}$$
(3.29)

Then, the ideal contact stresses $\sigma_{xa}(x_b, y_b)$ and $\sigma_{ya}(x_b, y_b)$ are compared to the vertical pressure distribution $\sigma_z(x_b, y_b)$. The vertical pressure distributions are taken from the semi-empirical tire contact patch model in Chapter 2, where $\sigma_z(x, y)$ was given for tires with $\varepsilon_{\alpha} = 0$. To account for carcass deformation, these distributions should instead be functions of x_b and y_b :

$$\sigma_z(x_b, y_b) = M \left(1 - r^n \cos^2 \theta - r^m \sin^2 \theta \right)$$
(3.30)

$$r = \sqrt{\left(\frac{x_b}{a}\right)^2 + \left(\frac{y_b}{b}\right)^2} \tag{3.31}$$

$$\theta = \tan^{-1} \frac{y_b/b}{x_b/a} \tag{3.32}$$

$$M = \frac{F_z}{\pi ab} \left(1 - \frac{1}{n+2} - \frac{1}{m+2} \right)^{-1}$$
(3.33)

Using a friction model, the resulting contact stresses $\sigma_x(x_b, y_b)$ and $\sigma_y(x_b, y_b)$ are determined. The friction model used in this thesis is similar to the friction model proposed by Sakai [45] and is based on a simple, two coefficient of friction model.

The composite ideal horizontal stress $\sigma_a(x_b, y_b)$ is divided by the vertical pressure

distribution $\sigma_z(x_b, y_b)$, resulting in the input value to the friction model. The output of the friction model, illustrated in Figure 3.9, is given by:

$$\sigma_{out}/\sigma_z = \begin{cases} \sigma_{in}/\sigma_z & \text{for } \sigma_{in}/\sigma_z \le \mu_a \\ \mu_s + (\mu_a - \mu_s) \exp\left[-\lambda \left(\sigma_{in}/\sigma_z - \mu_a\right)\right] & \text{for } \sigma_{in}/\sigma_z > \mu_a \end{cases}$$
(3.34)

with adhesion friction coefficient μ_a , sliding friction coefficient μ_s , and friction decay rate λ . The first case corresponds to adhesion: if the input value is less than the adhesion friction limit μ_a (1.6 in Figure 3.9), then the output from the friction model is the same as the input. The second case corresponds to sliding: if the input value is more than the adhesion friction limit μ_a , then that part of the contact patch is in sliding and the friction model will decrease the resulting value. This is modeled using an exponential decay that asymptotally approaches the sliding coefficient of friction μ_s (0.7 in Figure 3.9) at high input friction values.



Figure 3.9: Illustration of the friction model with adhesion limit $\mu_a = 1.6$, sliding limit $\mu_s = 0.7$, and friction decay rate $\lambda = 3$

The parameters chosen for Figure 3.9 are representative of a typical rubber compound on asphalt. Using the same friction model for a 1D brush tire model, approximately $\frac{F_y}{F_z} = 1.0$ is obtained using slip angle. The actual contact stresses $\sigma_x(x_b, y_b)$ and $\sigma_y(x_b, y_b)$ are found by scaling the ideal contact stresses $\sigma_x(x_b, y_b)$ and $\sigma_y(x_b, y_b)$ by the amount determined by the friction model. This gives the following:

$$\sigma_x(x_b, y_b) = \frac{\sigma_{out}(x_b, y_b)}{\sigma_{in}(x_b, y_b)} \sigma_{xa}(x_b, y_b)$$
(3.35)

$$\sigma_y(x_b, y_b) = \frac{\sigma_{out}(x_b, y_b)}{\sigma_{in}(x_b, y_b)} \sigma_{ya}(x_b, y_b)$$
(3.36)

This model is applied to a toroidal tire with a vertical pressure distribution as predicted by the linear deformation model (n = m = 2, see Section 2.3.2). The results for lateral slip angle are shown in Figure 3.10 and for camber angle in Figure 3.11. Both of these plots show the force distribution at their respective peak forces. Note the discontinuity near the rear of the contact patches. This represents the boundary of the adhesion region - the region of the contact patch behind this discontinuity is sliding. This discontinuity is sharper with slip angle because its ideal contact pressure continually increases toward the trailing edge. These figures illustrate clearly that camber makes better utilization of the available friction than slip angle.

3.4.1 Effects of Tire Carcass Deformation

In practice, the longitudinal carcass stiffness K_{cx} is quite high. For example, the data for the tire in Sakai [45] indicate that it is more than 10x larger than the lateral carcass stiffness K_{cy} . Therefore, its impact on tire model estimates is usually small.

For passenger car and motorcycle tires, the lateral carcass stiffness K_{cy} is often on the order of 100000 N/m. The resulting movement of the contact patch ε_y , sometimes known as pneumatic scrub [57], is on the order of 10 - 50 mm. One major effect of this is the overturning moment M_x generated by shifting the vertical force application F_z .

The torsional stiffness $K_{c\alpha}$ of passenger car and motorcycle tires is often on the order of 1000 Nm/rad, resulting in contact patch rotation ε_{α} on the order of $1^{\circ} - 2^{\circ}$. The *direction* of this rotation tends to be opposite for tire forces generated using slip angle versus camber.



Figure 3.10: Plot of lateral (σ_y) contact stress due to lateral slip angle α for the contact patch of a toroidal tire

Slip angle α generates a lateral tire force distribution that is centered behind the center of the contact patch, causing an aligning moment M_z that tends to turn the tire *out* of the turn. The aligning moment due to camber angle γ arises primarily due to the longitudinal deformation: rearward deformation on the inside of the tire and forward deformation on the outside generate an aligning moment M_z that tends to turn the tire *in* to the turn. Therefore, torsional carcass compliance has opposite effects for each: for slip angle, it causes the contact patch to twist out of the turn, effectively reducing its slip angle α and causing *less* lateral force demand, whereas for camber angle it generates a slip angle that causes *more* lateral force demand.

Therefore, to generate tire forces exclusively by camber, a small amount of slip angle must be added to compensate for the torsional carcass deformation ε_{α} . In fact, this is what was done for the plot in Figure 3.11.



Figure 3.11: Plot of lateral (σ_y) and longitudinal (σ_x) contact stress due to camber angle γ for the contact patch of a toroidal tire

3.5 Experimental Results

By using the prototype active camber suspension system developed in Chapter 5 on the rolling road described in Section 5.3.3, experimental data are taken using each of the three motorcycle tires described in Section 2.1. These are expressed as a series of tire curves (see Section 1.2.1) that plot steady-state lateral force F_y versus camber γ and slip angle α . These tire curves result from spinning the rolling road at 10 m/s and commanding the prototype suspension system to perform very slow ramp maneuvers that sweep across the steer and camber angle ranges needed.

As discussed in Section 1.2.1, brush tire models are expected to capture many, but not all of the effects illustrated in a tire curve. The typical expectations of the model for each region of the tire curve are given as follows:

• Low tire forces. At low levels of tire force, the tire force is approximately proportional to both slip angle and camber angle. This is often called the "linear" region of the tire, and the proportionality constant relating lateral

tire force F_y and the applied slip angle α and camber angle γ are given by cornering stiffness C_{α} and camber stiffness C_{γ} , respectively. These are both positive quantities and are defined as:

$$C_{\alpha} = -\frac{\partial F_y}{\partial \alpha}\Big|_{\alpha=0} \tag{3.37}$$

$$C_{\gamma} = -\frac{\partial F_y}{\partial \gamma}\Big|_{\gamma=0} \tag{3.38}$$

Brush tire models can be expected to perform well at low levels of tire force, accurately predicting the cornering and camber stiffnesses. In fact, the observed cornering and camber stiffnesses are typically used to determine the parameters of the brush model.

- Moderate/high tire forces (before peak). As the tire force is increased, more and more of the contact patch begins to slide. Therefore, the brush model prediction has an increasingly strong dependence on the sliding friction model used. For simple friction models, such as the one used in this thesis, the basic trends are often captured but the exact fidelity of the prediction is typically less than at low tire forces. More complicated friction models, such as the LuGre model [10], may capture this with greater fidelity.
- **Peak tire forces.** With the correct choice of friction coefficients, brush models can match the experimentally-observed peak friction value well. This is often used to determine the friction parameters in the brush model.
- Beyond the peak tire force. As slip angle or camber are increased beyond their values at the peak tire force, they typically exhibit a "roll-off" characteristic: the lateral force F_y , which has been increasing with camber and/or slip angle up to the peak, is now decreasing. This is because an increasingly large part of the contact patch is now sliding, and the role of the sliding friction model is increasingly important. Similar to the moderate/high tire force case, the basic trends are often captured but the exact fidelity of the prediction is typically less than at low tire forces or at the peak.

The resulting tire curves for camber γ and slip angle α are given in Figure 3.12 for the Dunlop tire, Figure 3.13 for the Avon tire, and Figure 3.14 for the Metzeler tire. The results of the Dunlop tire, specifically the observed cornering and camber stiffnesses, are similar to those of other, similarly-sized tires in literature (see Cossalter [8]). This suggests that the results obtained from the prototype suspension system are relatively accurate, and helps validate the measurements of the Avon and Metzeler tires for which there is little to no data in existing tire literature for comparison.

The top plots of each figure show tire curves for varying slip angle α with zero camber angle. Also included for the Avon and Metzeler tires are plots of combined steer and camber: a constant camber angle of $\gamma = 20^{\circ}$ is applied while slip angle α is varied. The results are consistent with the expectations given above. The model prediction at low slip angles matches data. As slip angle is increased, the lateral force begins to roll-off because an increasing portion of the contact patch is sliding. The simple friction model captures this basic trend, but not necessarily the exact values. Finally, as the lateral force approaches its peak, the model again matches data well.

The bottom plots of each figure show tire curves for varying camber angle γ with zero slip angle. As expected, the model predicts the behavior at low camber angles accurately. However, there is mismatch for the Avon and Metzeler tires above $\gamma = 20^{\circ}$ and for the Dunlop tire above $\gamma = 35^{\circ}$. This mismatch of roll-off behavior is not due to choice of sliding friction model; the friction limits are sufficiently high and have little effect even at $\gamma = 40^{\circ}$.

Rather, this mismatch is due to the tire sidewall. As observed in Section 2.2, the tire sidewall enters into and distorts the contact patch at high camber angles. The Dunlop tire, which intended for use on high-performance sports motorcycles, only begins to show this effect at high camber angles of about $\gamma = 40^{\circ}$. The Metzeler and Avon tires, which are intended for use on "chopper" custom motorcycles, show this effect even at moderate camber angles of $\gamma = 20^{\circ}$. Therefore, these results match expectation: operating the tire at conditions that cause sidewall distortion lead to degradation of tire performance.



Figure 3.12: Experimental and simulated tire curves for a Dunlop 180/55R17 tire on the rolling road for slip angle (top) and camber angle (bottom)



Figure 3.13: Experimental and simulated tire curves for an Avon 300/35R18 tire on the rolling road for slip angle (top) and camber angle (bottom)



Figure 3.14: Experimental and simulated tire curves for a Metzeler 300/35R18 tire on the rolling road for slip angle (top) and camber angle (bottom)

			Dunlop	Avon	Metzeler
model parameter		180/55 R17	300/35 R18	300/35 R18	
tire effective radius	r_{te}	m	0.312	0.325	0.325
tread profile radius	r_{tt}	m	0.105	0.200	0.200
inflation pressure	P	bar	2.4	2.4	2.4
normal load	F_z	Ν	2200	2800	2800
contact patch half-length	a	mm	53	51	51
contact patch half-width	b	mm	45	69	69
longitudinal tire band	m		1	2	2
compliance parameter	π		4	2	2
lateral tire band	m		1	0	10
compliance parameter	111		4	0	10
longitudinal brush stiffness	k_x	bar/mm	0.87	0.82	0.72
lateral brush stiffness	k_y	bar/mm	0.87	0.82	0.72
torsional carcass stiffness	$K_{c\alpha}$	Nm/rad	700	4000	2200
adhesion friction coefficient	μ_a		1.25	1.15	1.20
sliding friction coefficient	μ_s		1.25	1.15	1.20
friction decay rate	λ		3	3	3

3.5.1 Discussion of Model Parameters

Table 3.1: Table of brush model parameters for the Dunlop, Avon, and Metzeler tires

The parameters used for the three tires are given in Table 3.1. The tire radii r_{te} and r_{tt} are the same as those measured in Table 2.1. The normal loads F_z and inflation pressures P used for these tests are similar to the nominal conditions used for contact patch measurement in Chapter 2, allowing the contact patch parameters a, b, n, and m to be estimated accurately.

The brush stiffness k_y and torsional carcass stiffness $K_{c\alpha}$ are chosen to match the cornering stiffness C_{α} and camber stiffness C_{γ} of the tire. Increasing k_y has the effect of increasing both C_{α} and C_{γ} . However, as discussed in Section 3.4.1, increasing $K_{c\alpha}$ has the effect of increasing C_{α} and decreasing C_{γ} . Without data of tire performance due to longitudinal tire forces, it is difficult to estimate the longitudinal tire brush stiffness k_x directly; it was therefore assumed equal to the lateral brush stiffness k_y . The longitudinal and lateral carcass stiffness, K_{cx} and K_{cy} , affect only the model predictions of tire moments; because they have no effect on tire force predictions they are neither measured nor considered in this section.

All tires exhibit similar lateral brush stiffnesses k_y , but very different torsional carcass stiffnesses $K_{b\alpha}$. The lowest torsional stiffness is that of the Dunlop tire, which is designed for lower loads than the Avon or Metzeler and correspondingly is the lightest tire of the group. However, the torsional stiffnesses of the Avon and Metzeler tires, which are designed for similar loads and are of similar size and shape, are also different from one another. This reflects their carcass design: as discussed in Section 2.1 and observed in Section 2.4.2, the sidewalls of the Avon are much stiffer than those of the Metzeler.

The friction surface of the rolling road is not similar to that of asphalt road. One result of this is that the tires do not exhibit "roll-off" at high camber and slip angles. This indicates that the adhesion and sliding friction coefficients are nearly equal. Although each tire exhibited slightly different values, for each it was assumed that $\mu_a = \mu_s$ on the rolling road surface.

Figure 3.15 shows the model expectation for the Dunlop tire using friction values more indicative of an asphalt road, given by Figure 3.9 ($\mu_a = 1.6$, $\mu_s = 0.7$, $\lambda = 3$). The figure illustrates that the predicted gain in peak lateral force is small when using large amounts of camber (up to 40°) to supplement lateral force generation. This is consistent with the expectation in Section 1.2.1: motorcycle tires do not exhibit large gains in lateral tire force by using camber instead of slip angle. While existing motorcycle tires are useful for tire model and suspension prototype validation, they are not the correct tires for the final active camber concept. A specialized tire design is required.

3.6 Specialized Tire Design

3.6.1 Objectives

The main goal of this specialized tire design is to allow large lateral forces from camber on a car. This can be broken down into the following main objectives:

• Support for high camber angles. On any tire, increasing camber angle



Figure 3.15: Simulated tire curve for the Dunlop 180/55R17 tire using asphalt-like friction values

will eventually lead to significant contact patch distortion due to the sidewall. For car tires, this limit is typically a few degrees due to the relatively flat tread profile. For motorcycle tires, a curved tread profile increases this limit to around $40^{\circ} - 50^{\circ}$. The tread profile of the tire will also have curvature, allowing for high camber angles.

• High load rating. Typical sport motorcycle tires are designed to run with a nominal vertical load of about 1500-2000N with a maximum rating of about 3500N. However, the vertical load of the active camber vehicle will be over 3000N and will increase notably during handling maneuvers due to weight transfer. There are several car tires that can support these loads, but few motorcycle tires. Use of an under-specified tire may result in deteriorated performance and/or tire failure. Therefore, the tire needs to have a load rating similar to a car tire. To accomplish this, the tire needs to be sufficiently large and the contact patch sufficiently wide. While too much width is harmful to peak lateral camber force performance (see Section 3.3.3), too little will result in an overloaded contact patch.

- High peak lateral force from camber alone. The tire design should be optimized to provide the absolute maximum lateral camber force possible. This is accomplished by minimizing the amount of longitudinal contact stress in the contact patch, which erodes peak lateral force capability (see Section 3.3.3). This is affected by the basic tire geometry, but this is largely determined by the two first criteria. This is also affected by reducing the longitudinal brush stiffness k_x . This can be accomplished by tread pattern design.
- High peak lateral force from camber alone. The tire design should be optimized to provide the absolute maximum lateral camber force possible. This is accomplished by minimizing the amount of longitudinal stress in the contact patch, which erodes peak lateral force capability (see Section 3.3.3). Since wider contact patches generate more longitudinal stress, the tire should be made only as wide as needed to satisfy the load rating. Additionally, reducing the longitudinal brush stiffness k_x can help reduce longitudinal stresses, which can be accomplished by tread pattern design.
- High camber stiffness. This means that large lateral forces could be generated from small or moderate camber angles. Because there is an inherent constraint linking lateral force and lean angle on a motorcycle, motorcycle tires typically require about 45° of camber to develop 1g of lateral force. Since the automotive active camber concept does not have this constraint, a higher camber stiffness is desired. This would allow for reduced actuator movement, reducing packaging constraints and actuator requirements. This is accomplished by increasing lateral brush stiffness k_y .

3.6.2 Proposed Design

In developing the specialized tire design, ideally one would like each prototype to be a production-quality pneumatic tire. However, in practice, this may be prohibitively expensive and time-consuming. One possibility is to use solid tires for early prototypes. They have the advantage of being significantly less expensive to manufacture for prototyping purposes. While they certainly have many disadvantages for realworld use (noise, wear, heat, etc.), they are likely a useful step to validating this specialized tire design before investing in specialized pneumatic tires. Therefore, the proposed tire design illustrated in Figure 3.16 is a solid tire design.



Figure 3.16: Schematic of proposed specialized tire design

Below is a description of each part of the tire design and the principles applied to them:

• Overall size. The overall size is similar to a large motorcycle tire. It requires a rim size of 18" x 11.5", has a 690 mm overall diameter r_{te} , and a 310 mm overall width.

- Tread profile curvature. The tread profile curvature r_{tt} of 200mm creates a contact patch large enough to support the loads expected from an automotive active camber concept vehicle.
- Stiff base rubber. The stiff base rubber layer provides a stiff tire carcass, which will increase lateral brush stiffness k_y . This increases camber stiffness.
- Soft rubber. The soft rubber layer provides vertical compliance, similar to a pneumatic tire. This has the effect of decreasing vertical carcass stiffness k_z and increasing contact patch size. This increases camber stiffness.
- Tread rubber. The tread pattern is designed to reduce longitudinal stiffness k_x . This is accomplished by having several lateral grooves in the tire, but no longitudinal ones. This increases peak lateral camber force capability.
- **Circumferential cords**. These cords eliminate tire/rim slippage, and are used in several solid tire designs.

3.6.3 Simulation Results

The estimated parameters for the proposed tire design are given in Table 3.2. These parameters are used to generate many of the plots in previous sections. The vertical force distribution σ_z is assumed parabolic (n = m = 2 in the semi-empirical model of Chapter 2) and has size and shape as predicted by the physically-based tire model of Chapter 2. The lateral force distribution σ_y is given for slip angle α in Figure 3.10 and for camber angle γ in Figure 3.11.

The tire curves for the proposed tire design are given in Figure 3.17 using slip angle and Figure 3.18 using camber angle. The tire achieves a peak lateral force of 1.27gat a camber angle γ of 23°. This satisfies the requirements outlined in Section 3.6.1. As expected, the peak lateral force using slip angle α is lower, at just 1.00g.

One difficulty of this design is its large width. Although necessary to support the large weight of the test vehicle proposed in this thesis, it does suggest examining smaller, lighter-weight concept vehicles. These could make use of narrower tires, further increasing the benefits of active camber.

Specified Parameters			
Tire effective major radius	r_{te}	345	mm
Tread profile radius	r_{tt}	200	mm
Longitudinal brush stiffness	k_x	0.84	bar/mm
Lateral brush stiffness	k_y	1.68	bar/mm
Adhesion friction coefficient	μ_a	1.6	
Sliding friction coefficient	μ_s	0.7	
Friction decay rate	λ	3	
Effective tire inflation pressure	P	2.2	bar
Nominal vertical load	F_z	3600	Ν
Derived Parameters			
Contact patch half-length	a	83	mm
Contact patch half-width	b	64	mm
Contact patch area	A	167	cm^2
Contact patch aspect ratio	a/b	1.29	

Table 3.2: Table of estimated parameters for the proposed specialized tire design



Figure 3.17: Tire curve using slip angle α for proposed specialized tire design



Figure 3.18: Tire curve using camber angle γ for proposed specialized tire design

Chapter 4

Mechatronic Suspension Design

The active camber concept developed in this thesis requires a specialized, mechatronic suspension system. As described in Section 1.2.2, early in the design process it becomes clear that there is a significant challenge: while there are several, established suspension design criteria in literature for traditional suspensions, there aren't any for suspensions with multiple active degrees of freedom (DOF). Furthermore, from existing literature, it is unclear how traditional ideas might be adapted to accommodate the camber DOF.

The suspension system described in this thesis has a total of four degrees of freedom (DOF):

- Vertical suspension, sometimes referred to as jounce and rebound
- Steer
- Camber
- Wheel rotation

For purposes of developing suspension design criteria, all four of these DOF are considered active. The prototype developed in Chapter 5 includes active steer, active camber, and active vertical suspension DOF, but neglects active drive.

4.1 Overview

First, a physical description of the suspension system is given in Section 4.2. This details the basic layout of the active camber suspension and describes similarities and differences with respect to conventional suspension systems. It also describes some of the physical design constraints that have been imposed (e.g. actuator and joint types). This section is provided early in the chapter to give a physical reference for the abstract ideas generated in later sections.

Next, an analytic description of the suspension system is developed in Section 4.3. This is developed for generalized suspension systems as well as the specific cases of the 4DOF active camber and conventional suspension systems, making it possible to extend this method to different suspension designs. This section details different coordinate spaces and variables that are used to characterize the behavior of the suspension system. It also describes how a kinematic model of the suspension system can be used to generate mappings that relate these variables to one another, providing the basis for the suspension analysis in future sections.

The methodology used to derive design criteria for the suspension is developed in Section 4.4. It develops design *principles* by applying control and estimation objectives to the suspension system. The guiding philosophy behind this is that control and estimation laws can be simplified by careful mechanical system design.

Then, design criteria for the suspension are developed in Section 4.5. These result from applying the principles outlined in Section 4.4 to the mappings generated in Section 4.3, and are then related to specifications of the physical suspension described in Section 4.2. This section is focused on active camber suspension system design, but several notes are given on how this approach applies to conventional suspension systems as well as generalized suspension systems. The application to conventional suspension systems generates many of the same design criteria in existing literature, connecting the design method given in this thesis with existing literature.

Finally, the suspension system design is analyzed in Chapter 5 using the design criteria given in this chapter.

4.2 Basic Active Camber Suspension Layout

The basic suspension layout is given in this section to provide a visual reference to help interpret the abstract ideas developed in later sections. As such, it is presented with minimal design reasoning, which will be explained in Section 4.5 and Chapter 5.

4.2.1 Modularity with X1

The prototype suspension will be built on the X1 modular research vehicle platform, pictured in Figure 4.1. The modular platform allows multiple research projects to make use of similar components, drastically reducing the development time compared to a one-off research vehicle. In particular, it has separate front and rear suspension modules. This means that the complete active camber concept can be realized by adding specific suspension modules to the main X1 frame, which provides most of the infrastructure of an all-electric research vehicle.

The packaging constraints imposed on the suspension module design by the X1 platform are minimal. The mechanical interface between the two consists of a vertical-transverse mounting plane with two longitudinal frame rails. This is pictured in Figure 4.2 along with an example of a non-active-camber suspension module for X1.

4.2.2 Actuators and Joints

We are restricting the design to use only rotary actuators for vertical suspension, steer, and camber control. This allows the use of off-the-shelf components that only require electric power as opposed to hydraulics. This restriction does not appear to be a major limitation on suspension design. All actuators are fixed to the main vehicle frame, allowing simpler, more rigid supports and helping to minimize unsprung weight.

For drive control, the actuator is modeled as a rotary actuator acting on the wheel from the steering knuckle. This is representative of a typical (i.e. outboard) braking system or a hub-mounted electric drive motor. Since the initial suspension prototype has only a braking system and is not driven, this choice seems most appropriate. An



Figure 4.1: Picture of X1 with a non-active-camber suspension module



Figure 4.2: Schematic of X1 chassis (right) along with a non-active-camber suspension module (left), oriented with the front toward the left, illustrating the frame rails that attach to the suspension

alternative would be to act on the wheel from the vehicle frame, which is representative of an inboard braking system (not common on modern race or production cars) or a frame-mounted powertrain.

All joints in the suspension will be rigid. This is in contrast to production vehicles, where rubber bushings are frequently employed to provide a smoother ride. However, adding compliance increases uncertainty in suspension position measurement. This erodes controllability and observability, adding imprecision to control and inaccuracy to tire parameter estimation. Therefore, for this prototype, compliant joints will be omitted in favor of rigid ones.

4.2.3 Wheels and Tires

The first versions of the active camber suspension will make use of large motorcycle tires, described in Section 2.1. For purposes of this kinematic model, the contact surface of each tire is modeled as a rigid toroid, illustrated in Figure 4.3. The tire effective major radius is r_{te} and the tread profile radius is r_{tt} .



Figure 4.3: Diagram of tire toroid

4.2.4 Suspension Schematics

The basic active camber suspension layout resembles that of a modified dual control arm suspension. As such, it shares several things in common with conventional dual control arm suspensions.

Three orthographic views of the suspension schematic are given in Figure 4.4 and an isometric view with labels is given in Figure 4.5. These depict the suspension of one left wheel. Suspensions on right wheels would simply be the mirror image.

Each member is labeled with an acronym in the same color as the member itself. Each point is labeled with an acronym in orange with an arrow. Most of these points are joints, and their acronyms end with a "J". The joints illustrated with cylinders are revolute joints and those with circles are spherical joints. The four revolute joints that are grounded to the frame are illustrated with an additional bar in the cylinder.

Note that the lower control arm LCA and steering knuckle K are drawn schematically as triangles. All other members are simple, 2-force members and are therefore illustrated by straight lines.

There is symmetry with respect to the implementation of steer, camber, and vertical suspension actuation. Each degree of freedom consists of a rotary actuator fixed to the vehicle frame with an attached arm that connects to or near to the steering knuckle through an intermediate member. This results in a *parallel* mechanism design. In contrast to serial mechanisms, often employed for robotic arms and manipulators to permit larger workspaces, parallel linkages typically allow for higher rigidity. This is important since suspension systems are subject to large tire forces.

The reference axes shown are that of the vehicle body. Unless otherwise stated, any time in the remainder of this chapter that x-, y-, or z-axes are referenced, it is these three vehicle body axes. Their orientation follows ISO-8855. Where appropriate, the notation and orientation of other measurements in this thesis also follow this standard.

These diagrams are intended serve as a reference for the remainder of this chapter, and the acronyms that name them are used frequently. A description of each is given in the remainder of this section.





4.2.5 Steering Knuckle and Control Arms

Similar to a conventional dual control arm suspension, the steering knuckle (\mathbf{K}) is attached to the wheel (\mathbf{W}) and the control arms pivot to provide the knuckle with vertical suspension motion.

Members:

- K: Steering knuckle (in black). This is also known as the suspension knuckle or upright. Similar to conventional suspensions, this is the member that is attached to the wheel. Its position and orientation determine the position and orientation of the wheel.
- LCA: Lower control arm (in blue). This arm functions identically to a lower control arm in a conventional dual control arm suspension. This is advantageous because it is typically the most highly-loaded member in the suspension system. It is structurally easier to actuate other, more lightly-loaded members.
- UCA: Upper control arm (in magenta). This arm functions similarly to an upper control arm in a conventional dual control arm suspension, except that its inboard joint is moved to accomplish camber actuation (see Section 4.2.8).

Spherical Joints:

- LOBJ: Lower, outer ball joint. This attaches the lower control arm (LCA) to the steering knuckle (K). Because the active camber suspension requires large amounts of steering and camber actuation, this joint needs to allow a high misalignment angle.
- UOBJ: Upper, outer ball joint. This attaches the upper control arm (UCA) to the steering knuckle (K). Like the LOBJ, it should allow a high misalignment angle.

Revolute Joints:

- *LIBJ*: Lower, inner ball joint. This rotates around a near-longitudinal axis and attaches the lower control arm (**LCA**) to the vehicle frame.
- *UIBJ*: Upper, inner ball joint. This rotates around the longitudinal (x) axis and attaches the upper control arm (**UCA**) to the camber moment arm (**CMA**). In conventional suspensions, this joint attaches the upper control arm (**UCA**) to the vehicle frame.

4.2.6 Vertical Suspension Actuation

The vertical suspension motion is controlled by a relay linkage consisting of a moment arm (\mathbf{VMA}) and a pushrod (\mathbf{PR}) , pictured in magenta. To provide active control, an actuator is placed on the moment arm (\mathbf{VMA}) of this linkage.

Although not illustrated in Figures 4.4 and 4.5, the implementation also includes a passive, linear spring and damper assembly to reduce actuator effort. This eliminates the need for the actuator to expend effort simply to overcome gravity. This assembly is attached on one end to the vehicle frame and at the other end to an additional point on the moment arm (**VMA**).

Members:

- **PR**: Vertical suspension pushrod (in red). This pushrod functions similarly to a conventional suspension pushrod used to mount the spring/damper assembly inboard. It is a simple two-force member.
- VMA: Vertical suspension moment arm (in red). This is the arm that is controlled by the vertical suspension position actuator. It also has an additional attachment point for a passive spring/damper assembly to assist the actuator. Therefore, in implementation, it looks more like a bellcrank than a simple arm.

Spherical Joints:

- *LPRJ*: Lower pushrod joint. This attaches the vertical suspension pushrod (**PR**) to the lower control arm (**LCA**).
- UPRJ: Upper pushrod joint. This attaches the vertical suspension pushrod (**PR**) to the vertical suspension moment arm (**VMA**).

Actuated Revolute Joints:

• VAJ: Vertical suspension actuator joint. This rotates around the x-axis and attaches the vertical suspension moment arm (VMA) to the vehicle frame. The vertical suspension actuator acts on this joint.

4.2.7 Steering Actuation

Similar to conventional suspensions, steering motion is accomplished by allowing the steering knuckle (\mathbf{K}) to rotate around the outer control arm points (LOBJ, UOBJ). This motion is controlled by a relay linkage of a conventional tierod (\mathbf{TR}) that attaches to the steering knuckle (\mathbf{K}) on the outboard side and to the steering moment arm (\mathbf{SMA}) on the inboard side (pictured in green). The steering actuator acts on the steering moment arm (\mathbf{SMA}). The biggest functional difference between this and typical suspensions is that the left and right wheels are not connected: the steering actuation is independent.

Members:

- **TR**: Steering tierod (in green). This tierod functions similarly to a conventional suspension tierod used to connect the knuckle (**K**) to a steering rack or pitman arm. It is a simple two-force member.
- SMA: Steering moment arm (in green). This is the arm that is controlled by the steering actuator.

Spherical Joints:

- OTRJ: Outer tierod joint. This attaches the steering tierod (**TR**) to the steering knuckle (**K**).
- *ITRJ*: Inner tierod joint. This attaches the steering tierod (**TR**) to the steering moment arm (**SMA**).

Actuated Revolute Joints:

• *SAJ*: Steering actuator joint. This rotates around the z-axis and attaches the steering moment arm (**SMA**) to the vehicle frame. The steering actuator acts on this joint.

4.2.8 Camber Actuation

Camber motion is provided by moving the inboard side of the upper control arm (UCA). This is the biggest difference between the active camber concept and conventional suspensions. Similar to steering and vertical suspension position, it is controlled by a relay linkage (pictured in magenta). One link is the upper control arm (UCA) (see Section 4.2.5). The other is the camber moment arm (CMA), which is fixed to the vehicle frame on one end. The camber actuator acts on this arm via a simple parallelogram relay linkage. This relay linkage is necessary due to packaging: the camber actuator simply does not fit at the end of the camber moment arm (CMA). The design of this relay linkage is not important to the design criteria developed in this chapter.

Members:

- UCA: Upper control arm (in magenta). See Section 4.2.5.
- **CMA**: Camber moment arm (in magenta). This is the arm that is controlled by the camber actuator.

Spherical Joints:

• UOBJ: Upper, outer ball joint. See Section 4.2.5.

Revolute Joints:

• *UIBJ*: Upper, inner ball joint. See Section 4.2.5. Note that unlike the analogous joints in vertical suspension and steering linkages, this is a revolute joint, not a spherical joint. This is required to react longitudinal tire forces.

Actuated Revolute Joints:

• *CAJ*: Camber actuator joint. This rotates around the x-axis and attaches the camber moment arm (**CMA**) to the vehicle frame. The camber actuator is modeled as acting on this joint.
4.2.9 Drive Actuation and Wheel

The wheel shape used is the same as described in Section 4.2.3. The center of contact with the road is given by the contact point (CP). This is the modeled point upon which the tire forces act.

Drive/brake torque acts on the wheel from the steering knuckle \mathbf{K} . This is representative of a typical (i.e. outboard) braking system or a hub-mounted electric drive motor.

Members:

• W: Wheel and tire assembly (in cyan). This follows the geometry outlined in Section 4.2.3. It is attached to the steering knuckle (K).

Actuated Revolute Joints:

• WC: Wheel center. This rotates around the tire's y-axis, whose orientation varies with steer and camber angles. The drive actuator acts on this joint through the drive axle and constant velocity joints, which are not illustrated.

Other Points:

• *CP*: Contact point. In this model, this is the lowest point on the wheel/tire assembly (**W**), upon which the tire forces and moments act. In reality the tire is compliant and generates a contact patch, and this point is similar in meaning to the center of the contact patch.

4.2.10 Suspension Subframe

There are four suspension hardpoints illustrated in Figures 4.4 and 4.5, and all four are illustrated in black. These are the four revolute joints that attach the suspension members to the suspension cage (not illustrated), and three of them are actuated. The suspension cage is then fixed to the vehicle frame by a pair of framerails (See Section 4.2.1).

Revolute Joints:

• *LIBJ*: Lower, inner ball joint. See Section 4.2.5. This is implemented as a pair of revolute joints to handle large reaction moments.

Actuated Revolute Joints:

- VAJ: Vertical suspension actuator joint. See Section 4.2.6.
- SAJ: Steering actuator joint. See Section 4.2.7.
- CAJ: Camber actuator joint. See Section 4.2.8.

4.3 Description of Suspension Modeling Method

In control and estimation research, one often searches for the simplest model that is useful as an analytical tool. For suspension design, this is a static, rigid, kinematic model of one wheel's suspension. This model ignores dynamics, compliance, and the interaction of multiple wheels, greatly simplifying analysis. The method nevertheless provides valuable information about suspension performance.

Note that this static model is the same as a dynamic model that assumes that all suspension members are massless. Therefore, the criteria that are developed from the static model can serve as the basis for a more complicated dynamic analysis as well.

Also note that the method outlined here can be extended to *any* generalized suspension system design. Several notes are given in the following sections that pertain to the generalized case as well as the specific cases of some conventional suspensions and the active camber suspension.

4.3.1 Coordinate Spaces and Nomenclature

This method considers three sets of coordinate spaces for measurements of configuration (e.g. linear and angular positions) and effort (e.g. forces and moments) for each wheel's suspension.

They are:

- Tire space (\mathcal{T}) , with configuration variables q_t and effort variables e_t . This is the *output* space of the suspension.
- Suspension space (S), with configuration variables q_s and effort variables e_s . This is the *input* or *joint* space of the suspension.
- Vehicle space (\mathcal{V}) , with configuration variables q_v and effort variables e_v . This is the *reference* space of the suspension.

All of these use the vehicle frame as their basis for reference. This means that they have no dependence on the vehicle orientation in space (roll angle, pitch angle, etc.). This is different from several other definitions in which the vehicle orientation is implicitly included. For example, the tire space variables as defined in ISO-8855 relate the position of the tire to the road. For a given suspension position, this will change with the vehicle's roll and pitch angles. While useful for some dynamic vehicle simulations, definitions like these complicate an analysis intended only for suspension systems by including a dependence on vehicle orientation in space.

The vehicle frame axes definitions follow ISO-8855. Unless otherwise noted, all references to x-, y-, and z-axes in this thesis are made with reference to the vehicle frame axes. For many of the variables, although defined slightly differently from ISO-8855, the sign conventions are consistent with ISO-8855 standards where appropriate.

The configuration and effort variable sets are considered as vectors, with the following ordering:

$$q_{t} = \begin{bmatrix} h & \gamma & \delta & \theta_{d} \end{bmatrix}^{T}$$

$$e_{t} = \begin{bmatrix} F_{xt} & F_{yt} & F_{zt} & M_{xt} & M_{yt} & M_{zt} \end{bmatrix}^{T}$$

$$q_{s} = \begin{bmatrix} \theta_{va} & \theta_{ca} & \theta_{sa} & \theta_{da} \end{bmatrix}^{T}$$

$$e_{s} = \begin{bmatrix} \tau_{va} & \tau_{ca} & \tau_{sa} & \tau_{da} \end{bmatrix}^{T}$$

$$q_{v} = \begin{bmatrix} d_{x} & d_{y} & d_{z} \end{bmatrix}^{T}$$

$$e_{v} = \begin{bmatrix} F_{xv} & F_{yv} & F_{zv} & M_{xv} & M_{yv} & M_{zv} \end{bmatrix}^{T}$$

The following sections discuss and define each of these variables.

4.3.2 Tire Space

The tire space variables (\mathcal{T}) describe the configuration of and efforts on the wheel (\mathbf{W}) . From a suspension modeling standpoint, they characterize the *output* space of the system.

Configuration

In general, these variables should characterize the orientation of the wheel relative to the vehicle frame. There should be one variable for each configuration variable of interest for input into tire models, position control systems, etc.

For most suspension systems, this number is four. This includes both conventional suspension systems and the 4DOF active suspension. They are given as:

$$q_t = \begin{bmatrix} h & \gamma & \delta & \theta_d \end{bmatrix}^T \tag{4.1}$$

Three of these are the Euler angles that describe the *body-zxy* rotation from the vehicle frame to the wheel frame (δ , γ , and θ_d , respectively). The remaining one is a length (*h*). These are illustrated in Figure 4.6 and are given in their positive sign conventions.

- h: Vertical suspension position. This is the offset measured along the vehicle frame z-axis of the contact point (CP) from the nominal suspension position. Positive is upward wheel movement (jounce) and negative is downward wheel movement (rebound).
- γ: Camber angle. This is the x-axis Euler angle used in the rotation from vehicle to wheel frames. Note that this is different from another common definition of camber angle that defines positive as tilting the top of the tire outward from the vehicle. For left wheels, the two definitions have the same sign. For right wheels, they are opposite.
- δ : Steer angle. This is the z-axis Euler angle used in the *body-zxy* rotation from vehicle to wheel frames.
- θ_d : Drive angle. This is the y-axis Euler angle used in the *body-zxy* rotation from vehicle to wheel frames. Since tires are generally radially symmetric, this variable is often ignorable.

Note that in general, not all of these variables are independent. For example, in conventional suspension systems, camber angle (γ) is a function of the other configuration variables in q_t .



Figure 4.6: Schematic of tire configuration variables

Effort

In general, these variables should characterize the forces and moments acting on the wheel (**W**) at the contact point (CP). For all suspension systems, there are a total of six variables: three for the forces and three for the moments.

$$e_t = \begin{bmatrix} F_{xt} & F_{yt} & F_{zt} & M_{xt} & M_{yt} & M_{zt} \end{bmatrix}^T$$
(4.2)

The three force variables $(F_{xt}, F_{yt}, \text{ and } F_{zt})$ and three moment variables $(M_{xt}, M_{yt}, \text{ and } M_{zt})$ are measured with respect to the vehicle frame axes. Note that this is different from some other definitions which measure forces relative to an intermediate frame between the vehicle and wheel frames that has been rotated only by the steer angle (δ) and moments relative to either this frame or the wheel frame.

• F_{xt} : Tire force component aligned with the longitudinal vehicle axis.

- F_{yt} : Tire force component aligned with the lateral vehicle axis.
- F_{zt} : Tire force component aligned with the vertical vehicle axis.
- M_{xt} : Tire moment component aligned with the longitudinal vehicle axis. This is similar to, but not the same as the overturning moment.
- M_{yt} : Tire moment component aligned with the lateral vehicle axis. This is similar to, but not the same as the rolling moment.
- M_{zt} : Tire moment component aligned with the vertical vehicle axis. This is similar to, but not the same as the self-aligning moment.

Some of these variables are determined by tire dynamics. Therefore, in a complete simulation, some of these would be considered independent (e.g. F_{zt}) while others would be determined by a tire model (e.g. F_{xt} , F_{yt} , and M_{zt}).

4.3.3 Suspension Space

The suspension space variables (S) describe the configuration of and efforts on the suspension actuators. From a suspension control standpoint, they characterize the *input* space of the suspension. They are illustrated in Figure 4.7.



Figure 4.7: Schematic of suspension actuator variables

In general, the number of configuration and effort variables are equal to the number of DOF in the suspension system. Linear actuators would require one linear position and one force variable, and rotary actuators would require one angular position and one torque variable. Passive and active DOF have the same requirements on configuration and effort variables.

Since the suspension of the active camber concept has four DOF which all have rotary actuators, there are four angular positions for the configuration variables and four torques for the effort variables. Likewise, a conventional suspension with a linear spring/damper that is steered and driven (typical of a conventional front-wheel drive suspension) would have two angular positions, one linear position, two torques, and one force.

For active DOF, all of these variables are independent of one another. For passive DOF, the effort variable depends on its corresponding configuration variable and perhaps its derivatives. For example, the force exerted by a passive spring/damper assembly depends on the linear position and velocity that represent its elongation.

Configuration

In general, the configuration variables are measured as offsets from the nominal suspension position, which is when all independent tire configuration variables (q_t) are zero. If one of the tire configuration variables (q_t) is dependent on the others, as is the case with camber (γ) in a conventional suspension, its value is not necessarily required to define the nominal suspension position.

For the active camber suspension, all of these are angles. Positive values are given for rotations around positive axes in the vehicle frame. Since all four tire configuration variables (q_t) are independent, $q_s = 0$ when $q_t = 0$.

$$q_s = \begin{bmatrix} \theta_{va} & \theta_{ca} & \theta_{sa} & \theta_{da} \end{bmatrix}^T$$
(4.3)

- θ_{va} : Vertical suspension actuator angle. This is an angle about an axis parallel to the x-axis at the point VAJ.
- θ_{ca} : Camber actuator angle. This is an angle about an axis parallel to the x-axis at the point CAJ.
- θ_{sa} : Steer actuator angle. This is an angle about an axis parallel to the z-axis at the point SAJ.
- θ_{da} : Drive/brake actuator angle. This is an angle about the wheel's y-axis.

Effort

In general, these variables should characterize the efforts exerted by each actuator or passive suspension element.

For the active camber suspension, all of these are torques. Positive values are given for torques around positive axes in the vehicle frame, which is the same sense as their corresponding configuration variable.

$$e_s = \begin{bmatrix} \tau_{va} & \tau_{ca} & \tau_{sa} & \tau_{da} \end{bmatrix}^T$$
(4.4)

- τ_{va} : Vertical suspension actuator torque. This is a torque about the x-axis.
- τ_{ca} : Camber actuator torque. This is a torque about the x-axis.
- τ_{sa} : Steer actuator torque. This is a torque about the z-axis.
- τ_{da} : Drive actuator torque. This is a torque about the y-axis.

4.3.4 Vehicle Space

The vehicle space variables (\mathcal{V}) are defined with respect to the vehicle reference frame. The configuration variables characterize the offset from the center of gravity (CG) to the contact point (CP). The effort variables characterize the forces induced onto the vehicle at the CG by the suspension. From a suspension control standpoint, they characterize the *reference* space of the system. They are illustrated in Figure 4.8.



Figure 4.8: Schematic of vehicle space

For every suspension type, there are three configuration variables and six effort variables. The configuration variables are not necessarily independent of one another, but the effort variables are.

Configuration

The three configuration variables are measured as offsets along the vehicle axes from the vehicle CG to the contact point (CP). The sign convention follows that of a position vector from the CG to the contact point (CP).

$$q_v = \begin{bmatrix} d_x & d_y & d_z \end{bmatrix}^T \tag{4.5}$$

- d_x : Longitudinal offset.
- d_y : Lateral offset.
- d_z : Vertical offset. Note that this is always negative so long as the vehicle's CG is above the ground.

Effort

The six effort variables are the forces and moments induced on the vehicle frame at the CG by the suspension. They are measured along the vehicle axes.

$$e_v = \begin{bmatrix} F_{xv} & F_{yv} & F_{zv} & M_{xv} & M_{yv} & M_{zv} \end{bmatrix}^T$$
(4.6)

- F_{xv} : Induced longitudinal vehicle force.
- F_{yv} : Induced lateral vehicle force.
- F_{zv} : Induced normal vehicle force.
- M_{xv} : Induced roll moment.
- M_{yv} : Induced pitch moment.
- M_{zv} : Induced yaw moment.

4.3.5 Mappings

Mappings characterize the dependencies of variables in one coordinate space on those of another. Kinematic mappings relate the configuration variables to one another and Jacobians relate the effort variables to one another. They are derived from a complete kinematic model of the suspension system.

For example, the variables as defined in vehicle space (\mathcal{V}) are dependent on those in tire space (\mathcal{T}) . The tire configuration (q_t) uniquely determines the vehicle configuration (q_v) . Once the configuration is known, the tire efforts (e_t) uniquely determine the vehicle efforts (e_v) , too. These dependencies are encoded into the kinematic mappings and the Jacobians from tire space (\mathcal{T}) to vehicle space (\mathcal{V}) .

There are two mappings of each type (one forward, one inverse) between each pair of coordinate spaces, making a total of six mappings of each of the two types. However, not all of these mappings contain novel information. In fact, there are only two mappings of each type that need to be defined. From these, all others can be derived. As a matter of convention, these two are defined as those from the suspension space (S) to the tire space (T) and from the tire space (T) to vehicle space (V). These are the *forward kinematics* of the system. The Jacobians are given by the partial derivatives of these functions (which, depending on the specific coordinate definitions, may need to be rotated or translated after derivation).

The key idea of the method in this thesis is to use these mappings as the basis for suspension system analysis, from which design criteria for suspension systems are developed.

Kinematic Mappings

In general, the kinematic mappings may be nonlinear, transcendental, and complicated. They are represented as functions from one configuration vector to another and defined as:

$$q_t = f_s(q_s) \tag{4.7}$$

$$q_v = f_v(q_t) \tag{4.8}$$

These can be thought of as the *forward kinematics* of the system. The *inverse kinematics* are the inverses of these functions.

A diagram of how these mappings relate the configuration variables to one another is given in Figure 4.9.



Figure 4.9: Diagram of kinematic mappings of configuration variables

Jacobians

Unlike kinematic mappings, the mappings relating efforts to one another are completely linear. They do, however, depend on configuration. They are represented as matrices between effort vectors that depend on configuration and defined as:

$$e_t = J_s(q_s)e_s \tag{4.9}$$

$$e_v = J_v(q_t)e_t \tag{4.10}$$

where J_s and J_v are matrices representing the gains between the efforts.

A diagram of how these mappings relate the effort variables to one another is given in Figure 4.10. Note that [†] represents the psuedoinverse.



Figure 4.10: Diagram of Jacobian mappings of effort variables

The Jacobian for vehicle efforts $J_v(q_t)$ is square, and is given by:

$$\begin{bmatrix} F_{xv} \\ F_{yv} \\ F_{zv} \\ M_{xv} \\ M_{yv} \\ M_{zv} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & -d_z(q_t) & d_y(q_t) & 1 & 0 & 0 \\ d_z(q_t) & 0 & -d_x(q_t) & 0 & 1 & 0 \\ -d_y(q_t) & d_x(q_t) & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} F_{xt} \\ F_{yt} \\ F_{zt} \\ M_{xt} \\ M_{yt} \\ M_{zt} \end{bmatrix}$$
(4.11)

Since it has this structure, it always has an inverse which is the same matrix with the signs of the q_v variables $(d_x, d_y, \text{ and } d_z)$ inverted. This implies that with a given set of tire efforts e_t , the induced efforts on the vehicle CG e_v are always known. It also means that if the vehicle reaction efforts are known, the tire efforts can be determined uniquely.

4.3.6 Additional Notation Notes

The nominal suspension configuration is when all independent tire configuration variables (q_t) are zero. In the case of the active camber suspension, where all four of these are independent, the nominal configuration is when $q_t = 0$. If one of the tire configuration variables (q_t) is dependent on the others, as is the case with camber (γ) in a conventional suspension, its value isn't needed to define the nominal suspension position.

Some suspension design criteria deal with deviations of one variable due to variation in another variable. These are expressed using the following notation:

$$\Delta_y z(x,y) = z(x,y) - z(x,y)|_{y=0}$$

In this case, $\Delta_y z(x, y)$ denotes the deviation in z evaluated at some x and y from the value of z evaluated at the same x but with y = 0.

Some other suspension design criteria deal with one specific term in a Jacobian matrix. These are expressed similar to $J(q)_{yx}$, which represents the term in the Jacobian J(q) relating x and y.

4.4 Methodology for Design Criteria Generation

The basic idea behind this methodology is to analyze the mappings given in Section 4.3.5 using control and estimation design objectives. The guiding philosophy is to generate mechanical designs that simplify control and estimation laws.

What's special about this methodology is that all criteria for suspension design, from the conventional concepts to those specific to the active camber suspension, come from the same set of mappings. This makes the resulting criteria easier to support from first principles and easier to extend to novel suspension designs.

This section will focus on the active camber suspension in particular. However, this method is general enough to be applied to any type of suspension. For example, conventional suspension system design is used frequently in this section as a reference point.

4.4.1 Basic Control Objectives

To develop these criteria from control and estimation objectives, it is useful to first consider the basic control objectives. Two common types of basic control laws are position control and effort control. Position control is used when the primary control task is to maintain a specific position, and the effort required to do so is less important. Effort control is used in the opposite case, when the primary control task is to maintain a specific effort, and the position required to do so is less important.

This thesis asserts that the control objectives for most suspensions can be characterized in the same way. This is given by the following: suspensions typically intend to apply position control to steering and camber DOF, and effort control to drive and vertical suspension position DOF. This is because the output of the suspension is more readily characterized by camber and steer angles than by camber and steer torques, and by drive and vertical suspension efforts than by drive and vertical suspension positions.

For example, consider a conventional suspension with a passive spring/damper that is driven and steered. The driver's steering command to the steering wheel is likely characterized better by its angle than by its torque. The driver's drive command to the accelerator pedal is surely represented better by the resulting drive effort than by the drive angle (or the derivative of the drive angle, for that matter). The effect of the passive spring/damper on the output of the suspension is also likely characterized better by the resulting force than by the spring elongation. Finally, the effect of the mostly rigid links of the suspension on camber are better represented by the resulting camber angle than by the overturning moment.

This is also true with the active camber suspension. For steer and camber actuators, it will use position tracking controllers. This allows precise positioning of the tire for accurate lateral force determination. For drive and vertical suspension actuators, it will use torque control.

This is not to say that the other variables are unimportant. For example, steering torque, although arguably not the primary control variable, is very useful as a feedback signal to indicate how the tires are performing. Also drive angle, again arguably not often the primary control variable, is very useful as a feedback signal for anti-lock braking systems (ABS). The claim being made here is that while these are useful signals, they are not typically the *primary* variables being controlled.

The kinematic map $f_s(q_s)$ specifies the actuator positions q_s needed to command a given input position q_t . As such, it is the key modeling component needed for suspension position control, which is the basis for camber angle γ and steer angle δ control.

The Jacobian $J_s(q_s)$ specifies the gains from tire contact patch efforts e_t to suspension actuator efforts e_s . Therefore, it is the key modeling component needed for suspension effort control, which is the basis for vertical suspension h and drive θ_d control. These also can be used to feedforward additional torque terms for position control, improving performance.

4.4.2 Control Block Diagram

The control block diagram for the suspension system is given in Figure 4.11. As discussed in Section 4.4.1, position control is used for camber and steer actuators and effort control is used for the vertical suspension actuator. The effort control scheme

used for the drive actuator is omitted from the figure and not implemented in the prototype suspension system described in Chapter 5, but could be represented simply by the command torque being the product of the commanded longitudinal force and the effective tire radius.

The three commands are the commanded camber angle (γ cmd), the commanded steer angle (δ cmd), and the additional normal force (ΔF_z cmd). Correspondingly, the three outputs are the actual camber angle (γ act), the actual steer angle (δ act), and the estimated additional normal force (ΔF_z est). At the nominal suspension configuration, all of these are zero. As discussed in Section 4.2.6, a passive spring/damper assembly acts in parallel with the vertical suspension actuator and reacts the nominal normal force, requiring no actuator effort to maintain this nominal force. The additional normal force ΔF_z represents the *deviation* of the desired normal force from this nominal force.

The actual suspension system is represented by dark gray blocks. For control, each actuator is considered independently, implying that they are entirely decoupled. Of course, in the actual suspension system, the actuators are mostly but not completely decoupled. The coupling between actuators, as well as the effects of tire forces and the road profile, are represented as disturbances.

The only sensors used for control are the suspension encoders, providing the three actuator angles ($\theta_{ca} \text{ act}$, $\theta_{sa} \text{ act}$, and $\theta_{va} \text{ act}$). For each actuator, a command torque is calculated ($\tau_{ca} \text{ cmd}$, $\tau_{sa} \text{ cmd}$, and $\tau_{va} \text{ cmd}$), then scaled by the gear ratio and motor constant and sent as a current command to each motor controller.

Camber and steer control are similar. First, the inverse suspension kinematics (f_s^{-1}) convert the camber and steer angle commands (γ cmd and δ cmd) to camber and steer *actuator* angle commands (θ_{ca} cmd and θ_{sa} cmd). Then, feedback controllers use these angle commands along with the angle measurements (θ_{ca} act and θ_{sa} act) to compute the actuator torque commands (τ_{ca} cmd and τ_{sa} cmd) which are sent to the camber and steer actuators. Finally, the forward suspension kinematics (f_s) convert the resulting camber and steer actuator angles (θ_{ca} act and θ_{sa} act) to the actual camber and steer angles (γ act and δ act).



Because effort control is used for the vertical suspension actuator, it is very different from camber and steer. The inverse suspension Jacobian (J_s^{\dagger}) is used to map the commanded additional normal force $(\Delta F_z \text{ cmd})$ to a vertical suspension actuator torque command (τ_{va} cmd), which is sent directly to the actuator. Because the control system assumes decoupling between the three actuators, the (small) normal force reactions from the camber and steer actuators are ignored. However, the control system does include spring compensation. When the vertical suspension actuator is rotated (either by actuator torque or disturbance), it affects a displacement of the passive spring. Obviously, displacing a passive spring changes its spring force. This is represented as an additional vertical suspension actuator torque (τ_{spr} est), which is calculated from kinematics using the actual vertical suspension actuator angle (θ_{va} act). This is subtracted from the actuator torque command, effectively compensating for spring displacement. Because only the actuator angles are used for control, the estimated output additional normal force (ΔF_z est) is the same as the commanded input ($\Delta F_z \text{ cmd}$). Note that the control system does not compensate for the passive damper, which helps to ensure stability.

Although the inverse suspension kinematics (f_s^{-1}) are used only for converting camber and steer angle commands (γ cmd and δ cmd) to camber and steer actuator angle commands (θ_{ca} cmd and θ_{sa} cmd), they also require a vertical suspension position for calculation. For this, the control system uses the actual vertical suspension position (h act), which is calculated from the forward suspension kinematics (f_s) using the three measured actuator angles. The vertical suspension angle output from the inverse kinematics (θ_{va} cmd) is ignored.

4.4.3 Control and Estimation Objectives

This section describes which control and estimation objectives are applied to suspension systems to develop design criteria.

• Disturbance rejection. The major source of disturbance for a suspension system is road variation. Bumps in the road necessitate variation in vertical suspension position h. Therefore, by looking at how the maps depend on h, one

can assess how well the system deals with its major source of disturbance.

• Decoupled Control. The degree to which the control laws are coupled is an important consideration. This information is contained in the mappings that characterize the dependence of the primary control variables given in Section 4.4.1 between tire (\mathcal{T}) and suspension (\mathcal{S}) spaces. For highly decoupled control, the mappings should indicate that the primary control variable in the suspension space (\mathcal{S}) has a high dependence on its corresponding primary control variable in the tire space (\mathcal{T}) and low dependence on the other tire space variables.

For example, consider steering. Since it is considered a position control problem, the primary control variable is steer angle. To assess the coupling, examine the suspension kinematic mapping (f_s) from tire configuration variables (q_t) to suspension configuration variables (q_s) . For highly decoupled control, we want the steer angle actuator (θ_{sa}) to have a high dependence on the steer angle (δ) and a low dependence on the camber angle (γ) , vertical suspension position (h), and drive angle (θ_d) .

- Stability. To maximize control system design simplicity and safety, the suspension should be stable. Stability here is the behavior of the suspension as measured in q_t or q_s in the absence of control input e_s . Guaranteeing stability means that if the control system fails, the car will follow a straight trajectory. It also means that if the car is stopped and the control system is shut off, the suspension will stay near its nominal position. Sometimes external components are required to guarantee this, such as a parking brake on a car or a kick-stand on a motorcycle.
- Minimization of undesired effects. One such effect is tire scrub, which is the movement of the contact patch relative to the vehicle (d_x, d_y) during suspension movement. This increases tire wear and is likely to yield hard to predict tire forces.
- Response time. We'd like to execute dynamic driving maneuvers as quickly

as possible. This means bring able to react quickly to desired configuration inputs (q_t) , and the speed of this response is usually limited by actuator slew rate. It is best to design the system so that it is only limited by *one* actuator slew rate. That way, the control strategy is simple: saturate the input to that one actuator, then make the other actuators track the first actuator to achieve a smooth transition as measured in q_t . A system that did not have this property would be one where, when trying to implement the control strategy given above, the slew rate of another actuator is met. This makes control more complicated and performance less-predictable.

For the active camber suspension, dynamic driving maneuvers will typically require more movement in camber angle γ than in steer angle δ or vertical suspension position h. Therefore, it is desirable to ensure that the slew rate of the camber actuator ($\dot{\theta}_{ca,max}$) alone dictates the response time of the system.

4.5 Suspension Design Criteria

As discussed in Section 4.4, the suspension design criteria are derived from the mappings discussed in Section 4.3.5. Tables 4.1, 4.2, and 4.3 can be used to summarize this process. Each table represents one mapping with input variables at the top and output variables along the left. Each entry in the table is a suspension design criterion that relates a specific input to a specific output. For example, in Table 4.2, camber steer characterizes the dependence of the steer actuator angle θ_{sa} on the camber angle γ , which is derived from the kinematic mapping f_s , and is discussed in Section 4.5.3.

Table 4.1 represents the kinematic mapping f_v , given by Equation 4.8, that relates tire configuration q_t to vehicle configuration q_v . Tire scrub characterizes many of the dependencies in f_v . Note that by definition, d_z and h are 1 : 1.

Table 4.2 represents the kinematic mapping f_s , given by Equation 4.7, that relates suspension configuration q_s to tire configuration q_t . These are the forward kinematics of the suspension system. Each suspension actuator is designed to be the primary actuator for one tire configuration variable (e.g. the camber actuator primarily controls camber angle), as represented by the main diagonal in the table. The off-diagonal entries are characterized by bump camber, bump steer, and camber steer.

Table 4.3 represents the Jacobian J_s , given by Equation 4.9, that relates suspension efforts e_s to tire efforts e_t . The mappings from tire forces to the steer actuator and vertical suspension actuator depend strongly on the specific suspension design and give rise to several suspension design criteria. The mappings from tire forces to the drive actuator are determined by the effective tire radius, not the suspension design. The stability of the suspension system depends on the mappings from lateral and vertical tire forces to the camber and steer actuators. Each of the tire moments is reacted primarily by one actuator, represented by the ≈ -1 entries in the last three rows.

f_v		q_t				
		vertical				
		camber	steer	suspension	drive/brake	
		γ	δ	h	$ heta_d$	
	d_x	tire scrub (4.5.1)	tire scrub $(4.5.1)$	tire scrub (4.5.1)		
q_v	d_y	tire scrub $(4.5.1)$	tire scrub $(4.5.1)$	tire scrub $(4.5.1)$		
	d_z			1:1		

Table 4.1: Table of design criteria derived from the forward kinematic mapping from tire space to vehicle space (f_v)

f		q_s				
		vertical				
$\mid J$	S	camber	steer	suspension	drive/brake	
		$ heta_{ca}$	$ heta_{sa}$	$ heta_{va}$	$ heta_{da}$	
q_t	γ	primary actuator	camber steer (4.5.3)			
	δ		primary actuator			
	h	bump camber (4.5.2)	bump steer $(4.5.2)$	primary actuator		
	θ_d				primary actuator	

Table 4.2: Table of design criteria derived from the forward kinematic mapping from suspension space to tire space (f_s)

J_s		e_s					
		vertical					
		camber	steer	suspension	drive/brake		
		$ au_{ca}$	$ au_{sa}$	$ au_{va}$	$ au_{da}$		
	F_{xt}		scrub radius (4.5.10)	pitch center $(4.5.8, 4.5.9)$	effective tire radius		
	F_{yt}	stability $(4.5.11)$	mechanical trail (4.5.10), stability (4.5.11)	roll center (4.5.5, 4.5.6, 4.5.7, 4.5.9)			
e_t	F_{zt}	stability $(4.5.11)$	jacking moment arm (4.5.10), stability (4.5.11)	installation ratio and l_{va} (4.5.4)			
	M_{xt}	≈ -1					
	M_{yt}				≈ -1		
	M_{zt}		≈ -1				

Table 4.3: Table of design criteria derived from the Jacobian from suspension space to tire space (J_s)

4.5.1 Tire Scrub

It is desirable to minimize the dependence of d_x and d_y on all q_t variables, particularly on h. This has the effect of minimizing scrub, which is an undesired effect. This also has the effect of reducing variation in the Jacobian J_v . The dependence of d_z is fixed by the definition of h: it must increase or decrease in exactly a 1:1 ratio since h is the vertical suspension position.

So, the goal of kinematic map $f_v(q_s)$ design is to generate a map given by

$$\begin{bmatrix} d_x(q_t) \\ d_y(q_t) \\ d_z(q_t) \end{bmatrix} = \begin{bmatrix} d_x(0) \\ d_y(0) \\ d_z(0) - h \end{bmatrix} + \begin{bmatrix} \Delta d_x(q_t) \\ \Delta d_y(q_t) \\ 0 \end{bmatrix}$$
(4.12)

where $\Delta d_x(q_t)$ and $\Delta d_y(q_t)$ are minimized.

The deviations of $\Delta d_x(q_t)$ and $\Delta d_y(q_t)$ are largely determined by the design of the lower control arm (**LCA**) and the relationship between the outer joints of the lower and upper control arms (*LOBJ* and *UOBJ*). The line passing through these two points is the steer axis, which is the instantaneous axis of rotation of the wheel when subjected to steer actuation.

The relationship between the outer joints is characterized in the same manner as conventional suspensions. This is illustrated in Figure 4.12. Caster angle θ_c and kingpin angle θ_k describe the inclination of the steer axis from vertical. The mechanical trail t_m and scrub radius r_s describe the offset of steer axis from the center of the tire, the contact point CP, as measured at the ground.

The dependencies of Δd_x and Δd_y on vertical suspension position h are primarily determined by the length of the lower control arm (LCA). It should be as long as packaging allows. This design goal is similar to conventional suspension design.

The dependencies of Δd_x and Δd_y on camber angle γ are largely due to the vertical position of the outer joint on the lower control arm (*LOBJ*). In particular, it depends on the vertical distance between the lower, outer ball joint (*LOBJ*) and the point on the circle that defines the center of tire tread curvature (the dotted circle in Figure 4.3) directly above the contact point (*CP*). If *LOBJ* is directly on this



Figure 4.12: Diagram of steer axis and associated suspension parameters

point defining the center of curvature of the tire tread, then cambering will yield no tire scrub. However, there is another aspect to the height of LOBJ: it determines whether the suspension rises or falls slightly while cambering. If LOBJ is above the center of tread curvature, then cambering will cause the suspension to fall slightly. Similar to a motorcycle, powering down the suspension may cause the suspension to "fall over." If LOBJ is below the center of tread curvature, then the opposite occurs. Gravity stabilizes the suspension even when powered off. Note that typically, the range of possible positions is strongly limited by packaging.

The dependencies of Δd_x and Δd_y on steer angle δ are mainly due to the offsets of mechanical trail t_m and scrub radius r_s . The goal of minimizing dependence is accomplished by minimizing these two offsets for all suspension positions. This means making them small at their nominal position and making the kingpin and caster angles small so that their variation with suspension position is minimized. However, to ensure stability, mechanical trail t_m needs to be positive (see Section 4.5.11).

Therefore, the design objectives for the lower control arm (LCA) and outer control arm joints (LOBJ and UOBJ) can be summarized as:

- Make the lower control arm (LCA) as long as packaging allows.
- Set the nominal scrub radius r_s and kingpin angle θ_k to zero.
- Position the outer, lower ball joint (*LOBJ*) near to but slightly above the center of tread curvature.
- Provide only enough mechanical trail t_m to guarantee stability.

Note that, in general, a suspension design needn't have a simple steer axis defined by two joints. This is the case for several modern, multi-link suspension systems. However, there is always an instantaneous axis of rotation, and therefore still a virtual steer axis.

4.5.2 Bump Steer and Bump Camber

For disturbance rejection, it is desirable for tire forces to remain approximately constant in the presence of disturbances without the need to adjust actuator positions. This removes the need for the controller to compensate for disturbances explicitly.

The major source of disturbance for the suspension can be characterized by variation in vertical position h, which corresponds to variation in vertical suspension actuator position θ_{va} . The variables q_t that strongly influence tire forces are camber angle γ and steer angle δ . So, the dependence of the actuator angles that control these two values (θ_{ca} and θ_{sa}) on h should be minimized.

These dependencies are defined as bump camber $\Delta_h \theta_{ca}(q_s)$ and bump steer $\Delta_h \theta_{sa}(q_s)$, respectively and are given by:

$$\Delta_h \theta_{ca}(h, \gamma, \delta, \theta_d) = \theta_{ca}(h, \gamma, \delta, \theta_d) - \theta_{ca}(0, \gamma, \delta, \theta_d)$$
(4.13)

$$\Delta_h \theta_{sa}(h, \gamma, \delta, \theta_d) = \theta_{sa}(h, \gamma, \delta, \theta_d) - \theta_{sa}(0, \gamma, \delta, \theta_d)$$
(4.14)

Due to the radial symmetry of tires, these values have no dependence on drive angle θ_d .

The relative importance of bump steer vs. bump camber is in proportion to the tire forces they generate. Since cornering stiffness C_{α} is about an order of magnitude larger than camber stiffness C_{γ} (see Chapter 3, a given magnitude of bump steer $\Delta_h \theta_{sa}(q_s)$ is about an order of magnitude more important than the same magnitude of bump camber $\Delta_h \theta_{ca}(q_s)$.

Bump camber and bump steer are primarily determined by the camber and steer relay linkage designs (see Sections 4.2.7 and 4.2.8). The steer linkage design should place a very high importance on minimizing bump steer $\Delta_h \theta_{sa}(q_s)$. The camber linkage design, due to lower camber stiffness and additional design considerations (see Section 4.5.7), should place a much lower importance on minimizing bump camber $\Delta_h \theta_{ca}(q_s)$.

For conventional suspension system design, $\Delta_h \theta_{ca}(q_s)$ is known as camber gain. Because camber is actively controlled, camber gain is unimportant, and $\Delta_h \theta_{ca}(q_s)$ is more applicably considered as a disturbance of bump camber.

4.5.3 Camber Steer

As discussed in Section 4.4.3, it is desirable to have the camber actuator slew rate be the only one that limits system response speed to desired camber angle inputs. One way to help guarantee this is to ensure that, as camber angle γ is varied, the required angular change for the steer actuator θ_{sa} and vertical suspension actuator θ_{va} are small relative to the camber actuator θ_{ca} . This means that the dependence of the steer actuator θ_{sa} and vertical suspension actuator θ_{va} on γ should be small.

For reasonable suspension designs, the dependence of the vertical suspension actuator position θ_{va} on camber γ is already pretty small. However, reducing the dependence of the steer actuator θ_{sa} on camber γ requires additional design consideration. This dependence is defined as camber steer $\Delta_{\gamma}\theta_{sa}(q_s)$ and is given by:

$$\Delta_{\gamma}\theta_{sa}(h,\gamma,\delta,\theta_d) = \theta_{sa}(h,\gamma,\delta,\theta_d) - \theta_{sa}(h,0,\delta,\theta_d)$$
(4.15)

Similar to bump camber $\Delta_h \theta_{ca}(q_s)$ and bump steer $\Delta_h \theta_{sa}(q_s)$, this has no dependence on drive angle θ_d .

Camber steer can be reduced by placing the outer tie rod joint (OTRJ) on the steer linkage as low as possible on the knuckle (**K**). A near-ideal point would be at the same vertical height as the lower, outer ball joint (LOBJ), but this may be difficult due to packaging constraints with the wheel (**W**).

4.5.4 Installation Ratio and Vertical Suspension Actuator Effective Moment Arm Length

The installation ratio of a conventional, passive suspension system is defined as the unitless ratio of the suspension spring force over the normal tire force F_{zt} . For the 4DOF active suspension, which has a rotary vertical suspension actuator, similar information is given by the the relationship between the vertical suspension actuator torque τ_{va} and the normal tire force F_{zt} , which is represented by an effective moment arm length. This vertical suspension actuator effective moment arm length l_{va} can

be defined as:

$$l_{va}(q_s) = J_s(q_s)_{\tau_{va}F_{zt}} \tag{4.16}$$

For symmetric left and right suspensions, the sign of this arm length will be inverted due to sign conventions given in Section 4.2.4.

The vertical suspension actuator effective moment arm length l_{va} can be found in the Jacobian $J_s(q_s)$ as the term that relates input normal tire force F_{zt} to vertical suspension actuator torque τ_{va} . To simplify control and estimation laws, it is desirable to keep this gain relatively constant. This means minimizing the dependence of vertical suspension actuator effective moment arm length l_{va} on configuration q_t . For reasonable suspension system designs, the dependencies on all but the vertical suspension position h are already small.

Therefore, the design goal is to minimize the dependence of vertical suspension actuator effective moment arm length l_{va} on vertical suspension position h. This is accomplished by adjusting the locations of the joints in the vertical suspension linkage (VAJ, UPRJ, and LPRJ).

4.5.5 Roll Center

One important term in $J_s(q_s)$ is the gain between input lateral tire force F_{yt} and vertical suspension actuator torque τ_{va} . It determines how the vertical suspension effort must change when lateral force is applied. Looked at in another way, this encodes the same information as a force-based roll center (see [38] [35]).

As described in Section 1.2.2, the central idea behind roll center is that the application of lateral tire force F_{yt} not only induces a net lateral force to the vehicle F_{yv} of the same magnitude, but also may contribute an additional normal force reacted by other suspension links (not including the vertical suspension linkage) which is denoted as $\Delta_{Fyt}F_{zv}$. The ratio is given by:

$$\Delta_{F_{yt}}F_{zv} = -\frac{h_{rc}(q_s)}{d_y(q_t)}F_{yt}$$

$$(4.17)$$

where h_{rc} is the roll center height above the ground and d_y is the vehicle space variable that represents the lateral offset from the vehicle CG to the tire contact patch.

For a conventional, symmetric, non-steered, single DOF suspensions (ignoring the drive DOF) that are accurately depicted in 2D, this is exactly the same value as one would get from the conventional roll center construction (see Figure 1.20). This is found using the kinematic construction of finding the line that connects the instantaneous center of the control arms to the tire contact point. This kinematic construction effectively makes a new, instantaneous link for each wheel that transmits tire forces from the contact patch at one end to any point along the link, most commonly the point directly below the vehicle CG. The height of this point above the ground is the roll center height h_{rc} . The ratio of lateral to normal forces transmitted is given by the same ratio as in Equation 4.17.

However, once geometry is complicated by adding geometry that is not wellcaptured by a 2D representation and/or additional suspension links and DOF, it becomes unclear how to determine roll center by kinematic constructions. Therefore, the meaning of roll center is generalized by using the definition given in Equation 4.17. This still generates an effective point to translate tire forces, but it is no longer found using a simple kinematic construction. This new point is related to the force-based roll center concept [38] [35]. The primary difference is that Equation 4.17 defines the roll center using *active* forces instead of *reaction* forces. This facilitates extension to additional DOF and simplifies calculation.

For conventional suspension systems without too much added complexity, the two values of roll center are often very similar. For example, a steered, double control arm suspension found on the front of a typical car has a roll center that is usually wellapproximated by ignoring the steering linkage altogether and finding the conventional roll center as if it didn't have a steering DOF. In general, this approach is false: there are forces of non-negligible magnitude that are transmitted through the steering linkage that must be taken into account. However, conventional suspension systems of this type are also usually designed to minimize bump steer, so the steering tie rod is *also* directed at a point near the instantaneous center of the control arms. This means that the effective link created by the kinematic roll center construction is still a reasonable approximation, and the values are similar.

For conventional, passive suspension systems, this additional normal force on the vehicle F_{zv} will change the load on the passive suspension springs, causing the vehicle to move. The effect is opposite for left and right wheels (since the sign of d_y is inverted from left to right). This results in the car rolling. Because the lateral force is effectively applied to the vehicle at the roll center for each wheel, and because the roll centers are often similar for symmetric suspensions (except in extreme configurations), this roll motion is characterized by a moment arm between the roll center and the vehicle CG where the inertial reaction forces are. This leads to the name of "roll center."

As one might imagine, the implications of roll center on a 4DOF active suspension are different. Since the suspension is active, the roll motion can be arbitrary. In fact, if desired, it can be designed to mimic a passive suspension with a roll center of the designer's choosing. What's important is that to maintain a desired equilibrium position, this additional vertical force $\Delta_{F_{yt}}F_{zv}$ must be counteracted by an additional vertical suspension actuator torque. Therefore, it *couples* lateral and vertical suspension control. To enable a simpler, decoupled control structure, the roll center for the nominal suspension position will be on the ground: $h_{rc} = 0$.

This additional torque is specified by the gain in the map $J_s(q_s)$ from lateral tire force F_{yt} to vertical suspension actuator torque τ_{va} . It is given as

$$J_{s}(q_{s})_{\tau_{va}F_{yt}} = l_{va}(q_{s})\frac{h_{rc}(q_{s})}{d_{y}(q_{t})}$$
(4.18)

To make the roll center height zero, this gain also should be zero.

4.5.6 Roll Center Variation with Vertical Suspension Position

For disturbance rejection, it is desirable for the *effects* of roll center height to have a minimal dependence on vertical suspension position. These can be assessed by considering the effect roll center has on the roll mode of a vehicle. The analysis of many simple roll models used in vehicle dynamics studies show that it is the *distance* between the roll center and the vehicle CG that is important to the roll mode [28] [41] [6] [7] [20]. For passive suspension systems, keeping this distance constant has the effect of removing unwanted second-order effects from the roll response. Even with an active suspension, it is useful to consider the same design criteria. If the control inputs are held fixed and the vertical suspension position is allowed to vary, the deviation in suspension motion is just like the deviation of a passive suspension system. Therefore, for purposes of disturbance rejection, the 4DOF active suspension has the same design goal: maintain a constant distance between the roll center and vehicle CG. This means that the roll center height h_{rc} above the ground should move in a 1 : 1 ratio with vertical suspension position, effectively fixing it to the vehicle.

The amount by which roll center height h_{rc} changes with suspension position h is largely determined by the lengths of the two control arms. A wonderfully elegant method for estimating these lengths was developed by Maurice Olley who, at the time, desired to keep the roll center fixed in space [34]. It was later adapted to allow general n:1 roll center motion with vertical suspension position [41], where n is the ratio between vertical suspension position and roll center movement (as described above, the criteria used in this design is n = 1, moving the roll center 1 : 1 with the vertical suspension position). Since the lower control arm design is already determined by Section 4.5.1, this design method essentially determines the length of the upper control arm. Of course, it is incorrect as a final result, but remains a useful tool as a first approximation for design.

4.5.7 Roll Center Variation with Camber and Steer

For a given vertical suspension position h, it is desirable for the coupling between lateral and vertical control to stay approximately constant. This means that variation in roll center height h_{rc} with camber angle γ and steer angle δ should be minimized.

There is little additional design consideration needed to minimize the variation with steer angle. This is because, as with conventional suspension systems, this is
largely accomplished by minimizing bump steer $\Delta_h \theta_{sa}$ (see Section 4.5.2). There still may be a small amount of variation which might be tuned by altering steering linkage geometry. This would effectively be trading off bump steer $\Delta_h \theta_{sa}$ and the dependence of roll center h_{rc} on steer angle δ . Since small amounts of bump steer likely have a much larger impact on performance than small amounts of roll center height variation (which could be compensated using feedforward control if desired), this is entirely unnecessary: design the steering linkage to minimize bump steer and ignore dependence on h_{rc} by δ unless the suspension is some odd design for which this is abnormally large.

However, there is quite a bit more design consideration needed to minimize roll center height h_{rc} variation with camber angle γ . Since the lower control arm (LCA) and upper, outer, ball joint (UOBJ) are determined as in Section 4.5.1 and the upper control arm length by Section 4.5.6, this variation is largely determined by the position of the upper, inner ball joint (UIBJ) and how it moves to attain different camber angles.

For this, the kinematic-based roll center construction is a useful first approximation for design. The lower control arm (**LCA**) is determined as in Section 4.5.1. The desired roll center height h_{rc} is specified as in Section 4.5.5. This means that the location of the instantaneous center of the two control arms is also determined. This location, along with the upper, outer ball joint (*UOBJ*) on the upper control arm (**UCA**) specified by Section 4.5.1 and the approximate length of the upper control arm by Section 4.5.6, specifies the location of the upper, inner ball joint (*UIBJ*) of the upper control arm (**UCA**) for a given camber angle γ . Repeat this process for various camber angles, and they will approximately lie on an arc. Place the camber actuator at the center of that arc and the approximate design is complete. Iteration from this design should rely on looking at the change in the gain from lateral tire force F_{yt} to vertical suspension actuator torque τ_{va} in the Jacobian $J_s(q_s)$.

4.5.8 Pitch Center

Pitch center is a concept analogous to roll center, except that it deals with the vertical loads in suspension members induced by longitudinal tire forces F_{xt} . Its height above ground h_{pc} has an analogous impact on additional normal load, given by

$$\Delta_{F_{xt}}F_{zv} = -\frac{h_{pc}(q_s)}{d_x(q_t)}F_{xt}$$

$$(4.19)$$

Similar kinematic constructions can be made to approximate pitch center. Here, we consider instantaneous centers in a longitudinal-vertical plane. Generalizing this concept to a Jacobian-based pitch center results in Equation 4.19 above.

For passive suspension systems, a suspension's pitch center determines how the vehicle dives/squats when longitudinal force is applied to that suspension. This is what leads to the alternative names for pitch center parameters: anti-dive and anti-squat.

For the active camber suspension, pitch motion can be made arbitrary. Analogous to roll center, what's important is that to maintain a desired equilibrium position, this additional vertical force $\Delta_{F_{xt}}F_{zv}$ must be counteracted by an additional vertical suspension actuator torque. This is specified by the gain in the map $J_s(q_s)$ from longitudinal tire force F_{xt} to vertical suspension actuator torque τ_{va} , given as

$$J_s(q_s)_{\tau_{va}F_{xt}} = l_{va}(q_s)\frac{h_{pc}(q_s)}{d_x(q_t)}$$

$$(4.20)$$

To make the pitch center height zero, this gain also should be zero.

For an approximate design, we can again use the kinematic construction. This considers the axes about which the lower control arm **LCA** and upper control arm **UCA** rotate. When projected into the x - z plane, their rotation about the y-axis determines the location of the instantaneous centers of the kinematic construction. If both of these axes are horizontal when both arms are horizontal, as is the case at the nominal suspension position, then the pitch center is at the ground - $h_{pc} = 0$.

Also analogous to roll center, pitch center couples longitudinal and vertical suspension control. To enable a simpler, decoupled control structure, the pitch center for the nominal suspension position on the 4DOF active suspension will be on the ground: $h_{pc} = 0$. The same case made for simplifying the roll response can be made for pitch, motivating the desire to fix the pitch center to the vehicle.

The design parameters that control how pitch center moves with h are the orientation of the inboard axis about which the lower control arm **LCA** and upper control arm **UCA** rotate. Specifically, it is their rotation about the z-axis when projected into the x - y plane. For the active camber suspension system, the axis of rotation of **UCA** is moved to accomplish camber variation. To simplify design, it is desirable for this axis to remain parallel to the vehicle's x-axis. Therefore, 1:1 pitch center motion is designed by rotating the lower control arm **LCA**. The resulting instantaneous radii of rotation when projected into the x - z plane can be used in the same manner as control arm lengths in the design equations to control 1:1 movement (see [41]). Similar to roll center, this is incorrect as a final result, but remains a useful tool as a first approximation for design.

4.5.9 Instantaneous Centers and Screw Axes

The roll center is related to the instantaneous center due to vertical suspension motion in the y-z plane. In particular, the direction of the vector that points from the tire contact point CP to the roll center RC is the same as the vector that points to the instantaneous center in the y-z plane, given by components $(0, -d_y, h_{rc})$. This is the reasoning behind the conventional roll center construction (see Figure 1.20). Likewise, the direction of the vector that points from the tire contact point CP to the pitch center PC is the same as the vector that points to the instantaneous center in the x-z plane, given by the components $(-d_x, 0, h_{pc})$.

These concepts can be extended to 3D by considering the instantaneous screw axis of the vertical suspension DOF. This representation is similar to that of Suh [56]. The direction of the vector that is perpendicular to the instantaneous screw axis and passes through the tire contact point CP is related to the pitch and roll centers, given by the components $(-d_x/h_{pc}, -d_y/h_{rc}, 1)$. In fact, if a complete 3D model is used to find the instantaneous screw axis, the result is the same as calculation given by Equations 4.17 and 4.19.

4.5.10 Scrub Radius, Mechanical Trail, and Jacking Moment Arm

The terms in the Jacobian $J_s(q_s)$ that relate tire forces F_{xt} , F_{yt} , and F_{zt} to steering actuator torque τ_{sa} encode the same information as the conventional suspension design criteria of scrub radius, mechanical trail, and jacking moment arm respectively. These criteria are typically mapped from design principles using the concept of a steer axis (see Figure 4.12), and couple tire forces into steering actuator control.

To be precise, these terms are defined as the coupling of the forces as measured in axes attached to the tire that rotate with steer angle δ (but not camber angle γ). So, for non-zero steer angles, the coupling with the forces in e_t which are measured in the vehicle frame need to be corrected by a z-axis rotation equal to the steer angle δ . Also note that the more commonly used term "jacking torque" is the product of the jacking moment arm and the normal tire force F_{zt} .

For the active camber suspension, it is desirable to minimize scrub radius and jacking moment arm. This provides simpler, decoupled control laws. Scrub radius and jacking moment arm are determined by the positions of the outer joints on the control arms (LOBJ and UOBJ). The design criteria of tire scrub established in Section 4.5.1 accomplish the same goals, so there is no additional design consideration required. For mechanical trail design, it is important to consider its impact on stability.

4.5.11 Stability

It is important to consider the suspension design implications on stability. When stopped, the stability is determined by how h changes with q_s near the nominal suspension position $q_t = 0$. If h increases notably, then the vehicle CG falls and the suspension is unstable. If h increases only slightly, then stiction in the actuators and tire contact patch will likely stabilize the system. When moving, deviations in camber and steer angle generate additional tire forces which contribute to stability/instability. The stabilities of three of the four degrees of freedom are easy to guarantee for the stopped vehicle without additional design consideration. The passive spring placed on the vertical suspension moment arm (VMA) ensures that the vertical suspension actuator position θ_{va} is stabilized. For caster and kingpin angles that are not abnormally large, stiction ensures stability of the steering actuator position θ_{sa} . The stability of the drive actuator position θ_{da} is guaranteed either by relatively flat ground or a parking brake.

The stability of the camber actuator position θ_{ca} for the stopped vehicle requires additional design consideration. In the absence of stiction, it is only guaranteed if the lower, outer ball joint (*LOBJ*) is below the center of tire tread curvature (the dotted circle in Figure 4.3) directly above the contact point (*CP*). If *LOBJ* is below the center of tread curvature, then the suspension will rise slightly when cambered. Therefore, gravity stabilizes the suspension even when powered off.

At speed, one needs to ensure that tire forces generated by steer and camber angle deviations act to decrease the deviation. For steer angle δ , this is accomplished by making mechanical trail positive (see Section 4.5.10). This is the same criteria as conventional suspension design. For camber angle γ , this is guaranteed by the fact that the outer, lower control arm joint is above the ground.

Note that this only guarantees stability for *forward* speed. When driving backward, positive mechanical trail actually causes instability. Therefore, for safety, fast speeds in reverse should be avoided. The same is true for conventional suspension systems.

4.5.12 Condition Numbers and Well-Behaved Suspensions

Although not all of them are explicitly discussed in this chapter, every term in the Jacobian $J_s(q_s)$ has a specific meaning in suspension design. In broad terms, they determine the coupling between the input tire efforts e_t and the actuator efforts e_s . They determine the coupling between inputs and actuator efforts, which is useful for both control and estimation.

In general, it is desirous for these gains to be "well-behaved" in their variation

with configuration q_t . For specific gains, such as those discussed in the previous sections, this has a very specific meaning. For others, it may not. However, as a whole, the degree to which the gains are well-behaved can be measured by computing the condition number of the Jacobian matrix:

$$\kappa(q_s) = \operatorname{cond}\left(J_s(q_s)\right) \tag{4.21}$$

Note that since the gains in $J_s(q_s)$ have mixed units, the absolute values of the condition numbers $\kappa(q_s)$ aren't very meaningful. However, the way in which they change with configuration q_t is. If the condition number $\kappa(q_s)$ changes markedly near some q_t , then the suspension is likely not well-behaved near this position - it is heading toward a singularity. If it doesn't change very much, then it is a good indicator that the suspension system is likely well-suited to accurate control and estimation.

Chapter 5

Prototype Suspension Development

This chapter details the design and construction of the prototype active camber suspension system, which is based on the design criteria developed in Chapter 4.

As discussed in Section 1.2.2, one major challenge to suspension design is packaging. Put into perspective, a mechanism on the order of 1 m^3 in volume needs to provide 75° of camber movement, 30° of steer movement, and 100 mm of vertical suspension movement while rigidly supporting tire forces of up to 8000 N in each direction. As a result, the process of going from desired design criteria to a physical suspension design is not straightforward and often requires some degree of design compromise. One step-by-step process for tuning design criteria while navigating through packaging constraints to develop physical suspension design parameters is developed in Section 5.1. The resulting design criteria are presented and discussed in Section 5.2.

Finally, the implementation of this design is described in Section 5.3. This section includes illustrations of the completed suspension prototype. The completed prototype is attached to chassis dyno rollers (see Section 5.3.3). These rollers provide an experimental rolling road for testing, which is used to measure the performance of three different motorcycle tires. Discussion of the the suspension system performance observed in these tests in given in Section 5.3.5. The purpose of the prototype suspension is to be a completely flexible, capable experimental testbed for active camber research. As such, its specifications are a bit higher than a production active camber system might be. For example, the prototype suspension is designed to attain 45° of camber in one direction for motorcycle tire testing, but a production system with specialized tires (such as those hypothesized in Section 3.6) would not need this much range of camber motion. This would relax packaging constraints and allow the designer to have more freedom in fine-tuning the suspension design criteria. For this and other future suspension designs, the design criteria developed in Chapter 4 are still applicable. Furthermore, although this chapter is concerned primarily with the development of a specific prototype suspension system, the design process it presents still serves as a good model to follow for future suspension designs.

5.1 Kinematic Suspension Design Process

The purpose of this section is to describe the method used to design a suspension system with the layout given in Section 4.2. In contrast to more general principles, this shows the rationale behind the specific suspension design. It is intended as an aid to future suspension designers.

The method is described here as a series of steps. The idea is to proceed through the steps one at a time, generating an initial design. Then, once this initial geometry is developed, it can be iterated by adjusting the design parameters and checking results.

One of the key principles to keep in mind during this procedure is decoupling. As discussed in Section 4.4.3, one of the main goals of suspension system design is to make the different DOF as decoupled as possible. Not only does this simply control - it also simplifies the design process. This is because each DOF can be designed mostly independently, enabling a fairly straightforward, step-by-step design process.

Reading through this process will likely require frequent reference to Section 4.2 for diagrams and descriptions of the various members and joints of the suspension system, specifically to the labeled suspension schematic in Figure 4.5.

During the initial design process, the suspension was visualized not only with CAD but also by a scale model. A "construction set" for 1:2 scale suspension systems was built, shown in Figure 5.1. Quick and accurate fabrication of different suspension designs is facilitated by regularly-spaced holes and the use of a single thread size for all mounts, rods, and joints. For reference, included in the model (and shown at top left in Figure 5.1) is the X1 chassis framerails (see Section 4.2.1). Being able to move this scale suspension throughout its range of motion in 3D helped greatly in understanding packaging constraints prior to detailed design.



Figure 5.1: Scale model of suspension system (1:2 scale)

5.1.1 Step 1: Basic Constraints

Before designing the suspension, there are a few parameters that are already determined. These are the dimensions that describe the vehicle frame position at the nominal suspension position $(q_v(0))$ and the tire geometry. The vehicle frame dimensions are described by knowing the wheelbase, vehicle CG position and height, and track width. The tire geometry is characterized by knowing the two radii described in Section 4.2.3 $(r_{tt} \text{ and } r_{te})$.

5.1.2 Step 2: Outboard Control Arm Ball Joints

The first step is to determine the positions of the outboard ball joints on the steering knuckle (LOBJ and UOBJ).

First, the amount of nominal mechanical trail that is desired for stability is determined. This mechanical trail is determined by the combination of caster angle and offset (See Section 4.5.1). On many passenger cars this is accomplished by having a caster angle such that the steer axis projected in the side view (x - z plane) goes through the wheel center. However, this is not required. Because of packaging on the prototype active camber suspension, it is advantageous to get the required mechanical trail by using a zero caster angle.

It is desirable to have zero kingpin inclination angle and zero scrub radius. Doing so would place both of the outboard ball joint locations in the same x - z plane as the wheel center. However, due to packaging considerations, setting them exactly at zero isn't possible for the suspension prototype. However, they are close to zero; the kingpin angle θ_k is +1.7° and the scrub radius is +17mm at the nominal suspension configuration.

Next, the position of LOBJ along this steer axis is fixed. As discussed in Section 4.5.1, this should be slightly above the center of curvature of the tire tread. Since the caster angle is zero, this is slightly more than r_{tt} above the ground (see Section 4.2.3).

Finally, the position of the UOBJ along the steer axis is determined. This is not as critical or important as the LOBJ position. It should be placed fairly high inside the wheel, but the condition for it to be close to the center of tire curvature (as with the LOBJ) is relaxed. So, it is placed a little bit lower to simplify packaging.

5.1.3 Step 3: Control Arms

The length of the lower control arm **LCA** should be as long as packaging allows. This helps reduce tire scrub due to vertical suspension movement.

The length of the upper control arm **UCA** is determined by the desire for the roll center height (h_{rc}) to move 1:1 with vertical suspension position (h), effectively fixing

the roll center to the vehicle frame. This can be approximated using the kinematic roll center construction, and is approximated using the following equation (see [41]):

$$\frac{|d_y|}{h_{UOBJ} - h_{LOBJ}} \left(\frac{h_{UOBJ}}{l_{LCA}} - \frac{h_{LOBJ}}{l_{UCA}}\right) = n \tag{5.1}$$

where d_y is the lateral offset from the vehicle CG to the tire contact point, h_{LOBJ} and h_{UOBJ} are the nominal heights above ground of the lower and upper outer ball joints (found in Section 5.1.2), l_{LCA} and l_{UCA} are the lengths of the lower and upper control arms as projected in the y - z plane, and n is the desired n:1 movement of the roll center. As discussed in Section 4.5.6, this should be 1:1, so n = 1. This approximation is sufficient for the first design - later design iterations modified the values slightly to find the best geometry.

The inboard ball joint positions are determined by the desired roll center height (h_{rc}) at the nominal suspension position. As discussed in Section 4.5.5, this desired height is zero $(h_{rc} = 0)$ for decoupling. This is easily accomplished by making both control arms horizontal. Therefore, the nominal heights of the inboard ball joints are the same as the heights of the outboard ball joints.

Although the orientation of the inboard axis of the UCA is parallel to the x-axis, the inboard axis of the lower control arm LCA is rotated from this position about the z-axis. As discussed in Section 4.5.8, this accomplishes the goal of 1:1 pitch center movement with height variation. The design equations are similar to those in Equation 5.1:

$$\frac{|d_x|}{h_{UOBJ} - h_{LOBJ}} \left(\frac{h_{UOBJ}}{l_{LCA}} - \frac{h_{LOBJ}}{l_{UCA}}\right) = n \tag{5.2}$$

where d_x is the longitudinal offset from the vehicle CG to the tire contact point, h_{LOBJ} and h_{UOBJ} are the nominal heights above ground of the lower and upper outer ball joints, l_{LCA} and l_{UCA} are the lengths of the lower and upper control arms as projected in the x - z plane, and n = 1 to get the desired 1:1 motion. Note that because the **UCA** axis is parallel to the x-axis, $l_{UCA} = \infty$ for Equation 5.2. Similar to roll center, this design approximation is sufficient for the first design and was modified only slightly during design iteration to find the best geometry.

5.1.4 Step 4: Camber Moment Arm

The length of the camber moment arm (CMA) and position of the camber actuator joint (CAJ) are determined by the desire for the roll center height (h_{rc}) to be constant when camber angle is varied.

This is accomplished by determining exactly where the upper inner ball joint (UIBJ) should be positioned for each camber angle to keep the roll center height fixed. These positions lie approximately on an arc. The center of this arc is chosen as the CAJ location and the radius is the **CMA** length.

When the procedure in the previous sections is followed (where the roll center height is set at zero for the nominal suspension position and both control arms are horizontal) there is a simpler approximation. The position of CAJ is directly below UIBJ and directly inboard of LOBJ. This makes the **UCA** remain approximately horizontal as camber angle is varied, keeping the roll center height fixed at zero $(h_{rc} = 0)$.

5.1.5 Step 5: Vertical Suspension Linkage

The design of the vertical suspension linkage (VMA and PR) is largely determined by packaging and stress considerations. The only design criterion that needs to be considered here is the desire for a constant vertical suspension actuator effective moment arm length (l_{va}), which is the analog of installation ratio for a linear spring in a conventional suspension system (see Section 4.5.4).

The position of the lower pushrod joint (LPRJ) on the lower control arm LCA is determined first. Ideally, it should be approximately aligned with the centerling connecting the lower control arm ball joints (LOBJ and LIBJ) and as far outboard as packaging allows. Doing so generally reduces the forces in the members, allowing smaller, more rigid arms and simplifying packaging. However, the desire for the prototype suspension to permit +45° of camber makes this impossible. Therefore, LPRJ is placed behind the centerline of LCA, approximately halfway between the lower control arm ball joints (LOBJ and LIBJ). Then, the pushrod (**PR**) is inclined inboard from vertical only far enough to avoid interference with the tire/wheel as it cambers.

The design of the vertical suspension moment arm (VMA) is next. The angle between VMA and PR at the nominal suspension position should is approximately 90°. This helps to ensure that the vertical suspension actuator effective moment arm length (l_{va}) remains approximately constant. This is further aided by having a longer VMA. Note, however, that this length should not be too long. This is because the length of this arm affects the actuator effort required by the vertical suspension actuator - longer arms require more torque.

5.1.6 Step 6: Outer Tierod Joint

The position of the outer tierod joint (OTRJ) is determined by the desire to minimize camber steer (see Section 4.5.2). It is characterized by its height above ground and distance forward/rearward of the lower outer ball joint (LOBJ) at the nominal suspension position.

The outer tierod joint (OTRJ) is placed at the same height and lateral position as the lower outer ball joint (LOBJ) at the nominal suspension position. This eliminates camber steer at zero steer angle. To eliminate camber steer at nonzero steer angles, the distance forward/rearward of LOBJ should be very small. However, this constraint isn't reasonable: a very small forward/rearward distance between LOBJ and OTRJwould cause extremely high stresses in the steering linkage. Therefore, this distance is set at 83mm.

5.1.7 Step 7: Steering Linkage

The design of the tierod **TR** is determined by the desire to minimize bump steer. This procedure is the same as it would be for a conventional suspension system.

Since both control arms are approximately horizontal at the nominal suspension position, the tierod is also approximately horizontal. And, since the outer joints of the tierod (OTRJ) and lower control arm LOBJ are close to one another, the length of the tierod (**TR**) is the same as the lower control arm (**LCA**) as projected into the y - z plane.

For a well-behaved suspension, the length of the steering moment arm (SMA) is the same as the distance forward/rearward between the LOBJ and the OTRJ. This means that the gain from steer actuator angle to steer angle at the wheel is approximately constant and 1:1, which in turn makes the effort gains approximately constant. If it were in a different position, then at extreme steer angles, the gains would change.

For other suspension designs, the designer may decide that there should be some lateral offset between LOBJ and OTRJ for packaging reasons. If so, this offset should be approximately duplicated on **SMA** between SAJ and ITRJ.

5.1.8 Design Iteration

Once the initial suspension design is developed, some iteration is required to finetune the design criteria. For this purpose, software was written. The geometry of the suspension system was described in AutoLev, which was then used to generate code for MATLAB. This was used as the core solver of the program; pre- and postprocessing scripts were written around this solver in MATLAB. The result is the software described in Appendix B. The geometric parameters are given to the simulation program and it computes the design criteria. Then the designer can change the geometric parameters as desired and recalculate the design criteria.

The order of the steps in the procedure given above is important. The design of each step depends on the design of the previous steps. Therefore, one should *reverse* the order of these steps during design iteration. This means that any change made at one step of the design process given above requires changes for all steps *after* that one.

So, for a given design, first the tierod design should be adjusted to minimize bump steer (step 7). Then, the outer tierod joint location needs to be iterated to minimize camber steer (step 6). For each change here, the tierod design needs reiteration to again minimize bump steer. Once these two are dialed in, the designer should proceed backward one more step to the vertical suspension linkage (step 5). For every iteration here, the last two steps need to be reconsidered (steps 7 and 6). Once finished, the designer can proceed backward one more step to the camber moment arm (step 4). Again, for each iteration here, the last three steps regarding the vertical suspension and steering linkages require reiteration (steps 7, 6, and 5). This process continues all the way through the list of steps in the reverse order.

Here's an example of why this reverse process is needed: if the designer changes the control arm design to alter the roll center height (step 3), but skips the later step involving redesigning the steering linkage for bump steer (step 7), then the designer will draw incorrect conclusions as to the effect of their changes on roll center height. This is because the steering linkage, if not properly designed to minimize bump steer, will change the roll center height expected from control arm geometry alone.

5.2 Suspension Design Results

The suspension design was developed with the design criteria developed in Section 4.5 in mind. The results of this design process are given quantitatively in this section using the metrics detailed in Section 4.5.

Most of these metrics are scalar functions of three of the tire configuration variables (q_t) : camber angle (γ) , steer angle (δ) , and vertical suspension position (h). They are obtained numerically and are presented in plots. For each metric there are three plots, each one giving the values at a different steer angle (δ) . On each plot, there are four or five lines. For some metrics, these lines represent the values at different camber angles (γ) and are plotted against vertical suspension position (h). For others, the opposite is true: the lines represent the values at different vertical suspension positions (h) and are plotted against camber angle (γ) .

In all cases, the plots of these metrics are presented in such a way that the ideal case is represented by a single, flat, horizontal line. That is, the value of the metric as plotted should not vary with *any* of the configuration variables. Sometimes this ideal value is zero and sometimes it is not - this is specified in the section for each plot.

5.2.1 Range of Interest

The numerical analysis of the suspension system is restricted to the following ranges:

$$-0.05 \text{ m} \le h \le +0.05 \text{ m}$$
 (5.3)

$$-30^{\circ} \le \gamma \le +45^{\circ} \tag{5.4}$$

$$-15^{\circ} \le \delta \le +15^{\circ} \tag{5.5}$$

The asymmetry in the camber specification is due to packaging constraints. During development, it was found that designing a suspension to articulate all the way from -45° to $+45^{\circ}$ required too many design compromises.

This range is sufficient to characterize the overwhelming majority of desired driving maneuvers with the active camber concept. Perhaps the biggest exception is very low speed, high steer angle maneuvers, similar to those performed in a parking lot. Luckily, for the prototype active camber system, this isn't an important design consideration. Even when this is considered within the operating envelope, as is the case with a typical car suspension, the importance placed on fine-tuning design criteria at high steer angles on conventional suspension systems is still low. One artifact is that bump steer at these high angles is typically excessive, but does not represent a major hindrance to suspension performance.

5.2.2 Lower Control Arm and Outer Ball Joint Design

The specifications on the lower control arm (LCA) and outer ball joint (LOBJ and UOBJ) are determined by the design criteria given in Sections 4.5.1 and 4.5.11. They are largely restricted by packaging considerations, primarily of the brake rotor and wheel. The resulting specifications for the suspension are given in Table 5.2.2 below:

Table 5.1: Table of lower control arm and outer ball joint design parameters, measured at the nominal suspension position

Specification	Actual Value	Desired Value from Design Criteria
Length of LCA as projected in $y - z$ plane	$0.560 \mathrm{\ m}$	as large as packaging allows
Length of LCA as projected in $x - z$ plane	2.165 m	as needed for 1:1 h_{pc} movement
Height of $LOBJ$ below center of tire tread curvature	0.040 m	required for stability
Scrub radius at nominal suspension position	$0.016 { m m}$	as small as packaging allows
Mechanical trail at nominal suspension position	$0.043 \mathrm{~m}$	required for stability
Kingpin angle	1.7^{o}	as small as packaging allows
Caster angle	0.0^{o}	not specified

5.2.3 Tire Scrub and Jacking Moment Arm

Tire scrub (Δd_x and Δd_y) and jacking moment arm (l_{ja}) are defined and discussed in Sections 4.5.1 and 4.5.10. They are determined primarily by the specifications given in Section 5.2.2. Ideally, all of these values should be minimized.

The values of jacking moment arm length for the active camber suspension design are given in Figure 5.2. The tire scrub values are given in Figures 5.3 and 5.4. Keeping this value smaller at combined high steer and camber angles is difficult due to large compound angles. However, they are still acceptably small in the resulting design.



Figure 5.2: Jacking arm length l_{ja}







Figure 5.4: Tire scrub Δd_x

5.2.4 Bump Steer and Bump Camber

Bump steer $(\Delta_h \theta_{sa})$ and bump camber $(\Delta_h \theta_{ca})$ are defined and discussed in Section 4.5.2. They are determined primarily by the locations of the joints in the steering and camber linkages, respectively. Ideally, all of these values should be minimized. As discussed in Section 4.5.2, the relative importance of minimizing bump steer $(\Delta_h \theta_{sa})$ over bump camber $(\Delta_h \theta_{ca})$ is about an order of magnitude. This is because cornering stiffness (C_{α}) is about an order of magnitude larger than camber stiffness (C_{γ}) .

The bump steer and bump camber for the suspension design are given in Figures 5.5 and 5.6. The values of bump camber are sufficiently small to be considered a good design. Bump steer does increase a bit more at combined high steer and camber angles, but as discussed in Section 5.2.1, this is not too important of an operating region.



Figure 5.5: Bump steer $\Delta_h \theta_{sa}$



Figure 5.6: Bump camber $\Delta_h \theta_{ca}$

5.2.5 Camber Steer

Camber steer $(\Delta_{\gamma}\theta_{sa})$ is defined and discussed in Section 4.5.3. Like bump steer and bump camber in Section 5.2.4, it is determined primarily by the locations of the joints in the steering and camber linkages. Ideally, this value should be minimized, but it is impossible to make it zero. What is important is that, for a given camber angle (γ) , the camber steer $(\Delta_{\gamma}\theta_{sa})$ is notably smaller than the camber angle itself. This ensures that there will not be a problem with the slew rate limitation discussed in Section 4.5.3.

The camber steer for the suspension design is given in Figure 5.7. Because the value of camber steer $(\Delta_{\gamma}\theta_{sa})$ for a given camber angle (γ) is small relative to the camber angle itself, this is a good design.



Figure 5.7: Camber steer $\Delta_{\gamma} \theta_{sa}$

5.2.6 Vertical Suspension Actuator Effective Moment Arm Length

Vertical suspension actuator effective moment arm length (l_{va}) is defined and discussed in Section 4.5.4. Ideally, this value should remain constant for all suspension configurations.

The vertical suspension actuator effective moment arm length for the suspension design is given in Figure 5.8. It is sufficiently constant across suspension configurations.



Figure 5.8: Vertical suspension actuator effective moment arm length l_{va}

5.2.7 Roll and Pitch Center Heights

The roll center height (h_{rc}) is defined and discussed in Sections 4.5.5, 4.5.6, and 4.5.7. As discussed in Section 4.5.5, it should be zero at the nominal suspension configuration, change 1:1 with vertical suspension position (h), and have minimal dependence on camber angle (γ) and steer angle (δ) .

Pitch center is defined and discussed in Section 4.5.8. The requirements for pitch center height (h_{pc}) are very similar to those of roll center height: zero at the nominal suspension configuration, change 1:1 with vertical suspension position (h), and have minimal dependence on camber angle (γ) and steer angle (δ) .

Roll and pitch center height deviations from the desired 1:1 movement for the suspension design are given in Figures 5.9 and 5.10. These deviations are given by $h_{rc} + h$ and $h_{pc} + h$ to account for the *desired* change with vertical suspension position (h). Therefore, the values displayed in the plots should be minimized.

These plots, similar to some others before it, illustrate a difficulty in minimizing the criteria at combined large camber and steer angles. Much design iteration has reduced them to acceptable values, but some still remains. Nevertheless, the values calculated for the suspension design are acceptably small across suspension configurations.



Figure 5.9: Roll center height deviation from 1:1 movement $h_{rc} + h$



Figure 5.10: Pitch center height deviation from 1:1 movement $h_{pc} + h$

5.2.8 Condition Numbers

The condition numbers (κ) for the suspension Jacobian $(J_s(q_s))$ are discussed and defined in Section 4.5.12. For a "well-behaved" suspension system, they should stay relatively constant across different configurations.

These are given for the suspension design in Figure 5.11. They do not have a strong dependence on configuration, indicated a relatively good design. Note that, as discussed in Section 4.5.12, the measurement units for κ have little meaning and are therefore neglected.



Figure 5.11: Condition numbers of transfer matrices κ

5.3 Prototype Suspension System

5.3.1 Wheels and Tires

The first suspension prototype uses large motorcycle tires on a special set of wheels. They are shown in Figure 5.12.



Figure 5.12: The outside (left) and inside (right) of the wheels and tires used on the prototype suspension

Developing specialized tires for the active camber concept is costly and timeconsuming. Therefore, for the first prototype, existing motorcycle tires were used to test functionality and verify the tire model.

The wheels are a special, unique hybrid of a car and motorcycle wheel. The two wheels are identical to one another. They have the following specifications:

- Size. The wheel size is 18"x10.5", which supports the large motorcycle tires needed to sustain the vertical loads of the active camber concept.
- Center section. The wheel has a center section similar that of a typical car wheel. This gives it a normal hub with a lug bolt pattern.
- Wheel offset. Wheel offset is the distance from the center plane of the wheel to the hub mounting surface. The wheels acquired have a high offset of 53mm.

This large offset provides necessary room to package suspension and brake components. Typical motorcycle wheels have very little offset, so this is a great advantage for the suspension prototype.

• Rim design. The rim of the wheel is designed to be compatible with large motorcycle tires. As such, the bead and rim are designed to support the large loads required by the suspension prototype. Note that passenger car tires and motorcycle tires do not have compatible rim and bead dimensions - a wheel designed for one will not fit the tires designed for the other. This is why the special hybrid wheels are necessary.

Using these wheels, two different tires were tested: a Metzeler ME880 and an Avon Cobra. These are the same as used in the tire modeling studies of Chapters 2 and 3, and are described in Section 2.1.

5.3.2 Actuators and Sensors

The three actuators chosen are all brushless DC motors. They were chosen due to their low inertia properties relative to brushed DC motors. On each actuator is a gearbox.

The gearboxes on the steer and camber actuators have similar general requirements. Because steer and camber are considered position control problems (see Section 4.4.1), it is important that the gearbox has very high stiffness and essentially zero backlash. For this reason, 80:1 harmonic drives are chosen.

The gearbox on the suspension actuator, however, has very different requirements. Because suspension is considered an effort control problem, it is important that the gearbox/motor combination has very low inertia. This is equivalent to the concept of minimizing unsprung mass on a conventional suspension. The reflected inertia of the gearbox/motor combination as measured at the wheel adds to the unsprung mass. For this reason, a 20:1 planetary gearbox is chosen.

Each of the motors has an incremental encoder for noise-free position measurement. To get absolute position, each actuator has attached to its output arm a linear potentiometer. These are used during initialization to zero-out the encoder readings.

The discrete-time position controllers used for camber and steer are given in Figure 5.13. They consist of a feedback PD controllers with feedforward inertia compensation and filtered derivatives. The complete suspension system control structure into which these position controllers fit is given in Figure 4.11.

As discussed in Section 4.2.6, a spring and damper assembly is placed in parallel with the suspension actuator. This reduces the loads on the suspension actuator; the basic task of reacting gravity is taken by the spring. The damper is tuned to give a linear response so its effects are easily predicted in suspension control and estimation models.

One goal of the suspension system is to have on-board tire force estimation capabilities. For this purpose, the system has four load cells. They are placed:

• In the camber relay link. This gives a measurement of the camber actuator effort.



Figure 5.13: Block diagram of position controllers for camber and steer

- In the steering tierod. This gives a measurement of the steering actuator effort.
- In the suspension pushrod. This gives a measurement of the suspension actuator effort.
- At the rear, inner ball joint of the lower control arm LCA, affixed to the frame. Unlike the three members above, the LCA is not a simple 2-force member. It has a revolute joint on its inboard side, implemented with a pair of ball joints. With this arrangement, the reaction force components of the ball joints which are aligned with the rotation axis are statically indeterminate. Therefore, it is important that the load cell measurement axis is perpendicular to the LCA rotation axis so that it is insensitive to these forces.

By design, the three actuators have little to no coupling with longitudinal tire forces F_{xt} . Therefore, the first three load cells have little sensitivity to longitudinal tire forces F_{xt} . This is solved by placing the fourth load cell on the lower control arm, which reacts much of the longitudinal forces. The resulting sensor suite is now sensitive to longitudinal tire forces.

Using these four load cells, along with a kinematic model of the suspension, it is possible to calculate four of the six tire efforts independently. For the steady-state maneuvers in Chapter 3, it is assumed that $M_{xt} = M_{yt} = 0$ and the load cells are used to estimate F_x , F_y , F_z , and M_z .

Gravity compensation is required to get accurate tire force estimates from the load cells. The effect of gravity on the load cells is approximated by a model of a single point mass at the center of the wheel.

Housed inside the wheel are a braking system and a wheelspeed sensor. The braking system uses a conventional rotor and hydraulic caliper to enable tests of longitudinal tire forces. Also for use in longitudinal tire tests, the wheelspeed sensor gives an indication of the wheel's rotational speed.

5.3.3 Rolling Road

The primary use of the prototype suspension is as a tire tester. Therefore, a chassis dyno is used to simulate a rolling road. The rollers of this chassis dyno have a diameter of 1.2 m, as pictured in Figure 5.14. An encoder is placed on the roller to give a reading of the absolute simulated road speed.

As described in Section 4.2.1, the suspension subframe is designed to fit as a module on X1. To affix the suspension to the chassis dyno, an additional frame is built to hold this subframe to the ground. It has provisions for height adjustment to permit tests at different suspension positions.

5.3.4 Completed System

This section provides annotated pictures of the completed prototype suspension system:

- Figure 5.15 illustrates the main components.
- Figure 5.16 illustrates the camber components.
- Figure 5.17 illustrates the steer components.
- Figure 5.18 illustrates the suspension components and the LCA load cell.
- Figure 5.19 illustrates the in-wheel components.



Figure 5.14: The chassis dyno used as a rolling road for testing

• Figure 5.20 pictures the suspension at its limits of camber and steer angle actuation, illustrating its range of motion.

5.3.5 Performance

As described in Section 1.2.2, the prototype suspension system is designed for a dual purpose. Its primary design purpose is as a high-performance prototype suspension system for the active camber concept. As such, it was designed for on-vehicle use using the modular by-wire research testbed X1 (see Section 4.2.1). Its secondary design purpose is as a tire tester, providing data from motorcycle tires that are used to validate the 2D brush tire model in Chapter 3.

As a suspension system, the prototype functions quite well. In particular, it exhibits:





Figure 5.16: Prototype suspension: camber components




Figure 5.18: Prototype suspension: suspension components



Figure 5.19: Prototype suspension: in-wheel components



Figure 5.20: Prototype suspension: range of motion

- High rigidity. Even at high loads, the measured compliance of steer and camber angles is on the order of 0.5° maximum.
- Accurate positioning. The high rigidity, combined with sufficiently large actuators and resolute encoders, ensures that the suspension system is able to position the tire accurately.
- Fast response time. The large actuators, combined with well-tuned controllers, provide very fast response times with essentially no overshoot. The maximum slew rate of camber and steer angles is about $180^{\circ}/s$.
- Excellent consistency. The prototype is sufficiently refined and debugged to provide consistent, repeatable data. The largest variability is in the tires themselves their friction properties change when they get notably hot.

Therefore, it succeeds in its primary design purpose.

As a tire tester, the suspension system is able to provide sufficient data to build tire curves for tires, which are used in Chapter 3. However, there are some effects that limit the ultimate use of the suspension as a tire tester in its current form:

- Hysteresis. When plotting tire curves taken from ramp maneuvers that sweep back and forth across the range of motion, the results have hysteresis loops. The size of these loops does not change appreciably for different ramp rates, suggesting that it may be due to stiction in the suspension system. Stiction in joints between the tire and the load cells will cause offsets in the estimated tire forces. Steady-state tire forces can be estimated by averaging the results each branch of the hysteresis loop (e.g. one branch from a sweeps of steer angle with both positive steer rate and one branch from a negative rate). This method is used to generate the plots in Chapter 3. However, this hysteresis makes it hard to measure transient responses accurately.
- Pneumatic scrub. As discussed in Section 3.4.1, lateral tire carcass compliance causes the contact patch to move sideways relative to the wheel. This offset is known as pneumatic scrub. The problem is that lateral tire force estimates

from the four load cells are sensitive to pneumatic scrub, yet they cannot accurately measure it. This is illustrated for a simplified suspension schematic in Figure 5.21. The Jacobian relating tire forces to load cells indicate that the camber load cell force F_{clc} is the measurement principally responsible for estimating lateral tire force F_y . Lateral tire forces cause an x-moment about the lower outer ball joint LOBJ, which is reacted by the camber motor and therefore sensed by the camber load cell. The problem is that a pneumatic scrub r will offset the normal force F_z , also generating an x-moment about the lower outer ball joint LOBJ. The contributions from these two moments, rF_z and aF_y , are nearly indistinguishable with the four load cells, causing some discrepancy in lateral tire force data.

• Inertia. Although not required for this thesis, a more sophisticated model of inertia than the one used for gravity compensation in Section 5.3.2 is likely required to estimate transient tire forces. This is because the force path from tire forces to load cell measurements is fairly long. If the tire forces were measured more directly, this problem could be simplified.

As future work, these three effects could be mitigated by redesigning the steering knuckle and installing a six-axis load cell between the hub and knuckle. It provides a more direct measurement of tire forces and moments, which should reduce hysteresis, measure pneumatic scrub, and simplify the inertia model needed to capture fast transients.



Figure 5.21: Simplified schematic of prototype suspension, showing the lateral force path from the tire to the camber actuator

Chapter 6

Conclusion

The active camber concept developed in this thesis aims to increase the maneuverability of a vehicle by increasing its maximum steady-state turn rate. This is accomplished by increasing the maximum lateral tire force capability. Because camber makes better utilization of available friction in the contact patch than steer, camber can provide a notable increase in lateral tire force and therefore maximum turn rate and vehicle maneuverability. This thesis has taken several steps toward the realization of an active camber concept for extreme maneuverability.

Chapter 2 develops and validates a new model for the 2D shape and vertical pressure distribution of the tire contact patch. When a curved-profile tire is used to allow high camber angles, similar to existing motorcycle tires, the resulting contact patch shape and vertical pressure distribution requires a 2D representation. To illustrate this, contact patch measurements for three different motorcycle tires at different operating conditions are presented. Observations of these measurements, as well as the derivation of two physically-based models based on common assumptions in previous brush model work, lead to the development of a new, semi-empirical contact patch model. This model is used to parameterize the three tires, successfully capturing the shape and vertical pressure distribution in all cases when the contact patch is not sufficiently distorted by the sidewall.

Chapter 3 extends a brush tire model to 2D to capture the effects of cambering tires. Brush tire models are a class of tire force models commonly used in vehicle handling models. Typically, these models are 1D and only consider pressure distribution variations in the longitudinal direction of the contact patch. While sufficient for conventional passenger car tires with slip angles, this is not a good approximation of a curved-profile tire with camber. Experimental data is presented from three different motorcycle tires, the same as used for contact patch measurements in Chapter 2. These data result from using the prototype suspension system on an experimental rolling road, and serve to validate the 2D brush tire model. Once validated, the model is used to give a clearer picture of how camber can utilize friction better than slip angle, and is used to develop a hypothesis of how a specialized tire for the active camber concept could provide 30% more peak lateral force from camber.

Chapter 4 presents a set of design principles and design criteria for mechatronic suspension systems. These principles are stated by using control and estimation objectives and applied to a complete, kinematic model of the suspension system. From this, design criteria are derived from the forward kinematics, inverse kinematics, and Jacobians of the suspension system. By applying them to conventional suspension systems, design criteria are developed that are similar to existing suspension design literature. By applying them to a suspension system with active camber, active steer, and active vertical suspension, design criteria for the active camber concept are developed.

Chapter 5 realizes a complete prototype suspension system for the active camber concept. A prototype suspension system for the active camber concept is developed using the design criteria developed in Chapter 4 while also negotiating other constraints imposed primarily by packaging. To do so, the thesis presents one method for stepping through the process of suspension design and analysis. The final design is constructed and implemented successfully, and is attached to chassis dyno rollers. This provides an experimental rolling road for testing, demonstrating the capability of the suspension system as a research testbed.

To develop further the active camber concept in the future, the work of this thesis could be extended by:

• **Specialized tire development.** Specialized tires should be developed that maximize lateral force specifically for the active camber concept, based on the

understanding gained from the validated tire model. The prototype suspension on the chassis rollers provides a capable apparatus for testing the capability of these new tires, and would be further enhanced by the addition of a six-axis load cell between the knuckle and hub (see Section 5.3.5).

- Full-vehicle development. After both the suspension and tire prototypes have been developed, the suspension system and specialized tires should be placed on all four corners of a test vehicle. Furthermore, the understanding gained during this thesis should be used to guide the basic layout and size of the vehicle itself. This provides a complete testbed for the active camber concept.
- Full-vehicle control strategy. Once the complete vehicle testbed is built, a cohesive strategy for controlling all four camber and steer angles to optimize the handling capabilities of the vehicle should be developed.

This thesis demonstrated the functionality of the suspension system and the validity of the tire model by using commercially-available motorcycle tires. While these tires did not demonstrate much of an advantage to active camber, they did demonstrate that the model used to predict camber performance is valid. Along with the F400's 28% increase in lateral force, this gives validity to the hypothesized tire design resulting from the tire model. Therefore, once these tires are constructed and implemented on all four corners of a test vehicle, the active camber concept should indeed provide the predicted 20-30% more cornering force than steer alone.

The result: a vehicle with more cornering capability than anything else on the road, a vehicle with extreme maneuverability.

Appendix A

Contact Patch Geometry Derivation

By using tire geometry, one can approximate the contact patch shape and vertical pressure distribution. An outline of the process is given as follows:

- Assume the road is flat and rigid. This means that the contact area between the tire and the road will be a flat plane. Also, this implies that deformation occurs only in the tire.
- Assume that only the inflation air carries the vertical load. This means that the contact patch area A can be approximated using only the normal force F_z and the inflation pressure P.
- Deform the tire into the road until the correct contact area is obtained. This deforms the tire vertically by a distance h, generating a flat contact patch at its bottom. The resulting vertical deformation profile is given by $\delta_z(x, y)$.
- Determine the vertical pressure distribution from the vertical deformation. It is assumed that the vertical pressure distribution $\sigma_z(x, y)$ is proportional to the vertical deformation $\delta_z(x, y)$. By equating the total force

 F_z with the integral of the vertical pressure distribution, the vertical stiffness k_z used to relate $\sigma_z(x, y)$ and $\delta_z(x, y)$ can be found.

This process can be applied to tires of any geometry. Of particular interest is toroidal geometry, which is representative of motorcycle and active camber concept tires. This appendix uses this tire geometry to illustrate that the contact patch shape is well-approximated by an ellipse and the vertical pressure distribution $\sigma_z(x, y)$ is well-approximated by a paraboloid.

The first part of the derivation approximates the shape of the toroidal tire in the region of the contact patch by a paraboloid. This approximation assumes that the vertical deformation is small relative to the tire radii (r_{te} and r_{tt}). Then, this approximation is used to derive analytic equations for the contact patch shape and vertical pressure distribution. Finally, the validity of this approximation is illustrated by comparing its results against numerical results using the true toroidal geometry.

The results are indicative of the linear deformation model outlined in Section 2.3.2. For the balloon model, the only difference is that the vertical pressure distribution $\sigma_z(x, y)$ has no dependence on the vertical deformation profile $\delta_z(x, y)$ and instead is uniform. The paraboloid approximation of the toroid and its application to the elliptical contact patch shape are the same.

A.1 Paraboloid Approximation

The toroid tire shown in Figure 2.1 is symmetric about the y-axis with origin located at the center of the contact patch (as opposed to the center of the torus, as is more typical with torus representations). This torus can be expressed as a set of parametric equations ([19]). The three coordinates (x, y, and z) are defined by two parameters $(\theta \text{ and } \phi)$ and are given by:

$$x = (r_{te} - r_{tt} (1 - \cos\phi)) \sin\theta \tag{A.1}$$

$$y = r_{tt} sin\phi \tag{A.2}$$

$$r_{te} - z = (r_{te} - r_{tt} (1 - \cos\phi)) \cos\theta \qquad (A.3)$$

The parameters can be eliminated by combining these three equations into a single equation as follows:

$$x^{2} + (r_{te} - z)^{2} = (r_{te} - r_{tt} (1 - \cos\phi))^{2}$$
(A.4)

$$x^{2} + (r_{te} - z)^{2} = \left(r_{te} - r_{tt}\left(1 - \sqrt{1 - \sin\phi^{2}}\right)\right)^{2}$$
(A.5)

$$x^{2} + (r_{te} - z)^{2} = \left(r_{te} - r_{tt} \left(1 - \sqrt{1 - \frac{y^{2}}{r_{tt}^{2}}} \right) \right)^{2}$$
(A.6)

$$x^{2} + (r_{te} - z)^{2} = \left(r_{te} - r_{tt} + \sqrt{r_{tt}^{2} - y^{2}}\right)^{2}$$
 (A.7)

Solving for x^2 , Equation A.7 can be expressed as the difference of two squares. Then, these squares can be expanded, eliminating terms:

$$x^{2} = \left(r_{te} - r_{tt} + \sqrt{r_{tt}^{2} - y^{2}}\right)^{2} - (r_{te} - z)^{2}$$
(A.8)

$$x^{2} = \left[(r_{te}) - \left(r_{tt} - \sqrt{r_{tt}^{2} - y^{2}} \right) \right]^{2} - \left[(r_{te}) - (z) \right]^{2}$$
(A.9)

$$x^{2} = -2r_{te}\left(r_{tt} - \sqrt{r_{tt}^{2} - y^{2}}\right) + \left(r_{tt} - \sqrt{r_{tt}^{2} - y^{2}}\right)^{2} + 2zr_{te} - z^{2} \quad (A.10)$$

Because it is assumed that the contact patch size is notably less than the tire radii, $y \ll r_{tt}$. Therefore, the expression $r_{tt} - \sqrt{r_{tt}^2 - y^2}$ can be approximated by a parabola. The appropriate parabolic approximation is found by matching the second derivative evaluated at y = 0 of the function $f(y) = r_{tt} - \sqrt{r_{tt}^2 - y^2}$:

$$f(y) = r_{tt} - \left(r_{tt}^2 - y^2\right)^{\frac{1}{2}}$$
(A.11)

$$\frac{df(y)}{dy} = y \left(r_{tt}^2 - y^2\right)^{-\frac{1}{2}}$$
(A.12)

$$\frac{d^2 f(y)}{dy^2} = y^2 \left(r_{tt}^2 - y^2\right)^{-\frac{3}{2}} + \left(r_{tt}^2 - y^2\right)^{-\frac{1}{2}}$$
(A.13)

$$= r_{tt}^{2} \left(r_{tt}^{2} - y^{2} \right)^{-\frac{3}{2}}$$
(A.14)

$$\frac{d^2 f(y)}{dy^2}\Big|_{y=0} = \frac{1}{r_{tt}}$$
(A.15)
(A.16)

and the function $g(y) = my^2$ at y = 0:

$$q(y) = my^2 \tag{A.17}$$

$$g(y) = my^{2}$$

$$\frac{dg(y)}{dy} = 2my$$
(A.17)
(A.18)

$$\frac{d^2g\left(y\right)}{dy^2} = 2m \tag{A.19}$$

$$\left. \frac{d^2 g\left(y\right)}{dy^2} \right|_{y=0} = 2m \tag{A.20}$$

(A.21)

The result is:

$$m = \frac{1}{2r_{tt}} \tag{A.22}$$

$$r_{tt} - \sqrt{r_{tt}^2 - y^2} \approx \frac{y^2}{2r_{tt}}$$
 (A.23)

Using the approximation given in Equation A.23, Equation A.10 can be simplified as follows:

$$x^{2} = -2r_{te}\frac{y^{2}}{2r_{tt}} + \left(\frac{y^{2}}{2r_{tt}}\right)^{2} + 2zr_{te} - z^{2}$$
(A.24)

$$x^{2} = -\frac{y^{2}}{2r_{tt}} \left(2r_{te} - \frac{y^{2}}{2r_{tt}}\right) + z \left(2r_{te} - z\right)$$
(A.25)

Because the tire radii are much larger than the deformation amounts, $z \ll r_{te}$ and $\frac{y^2}{2r_{tt}} \ll r_{te}.$ Therefore, Equation A.25 can be simplified by:

$$x^{2} = -\frac{y^{2}}{2r_{tt}} \left(2r_{te} - 0\right) + z \left(2r_{te} - 0\right)$$
(A.26)

$$x^{2} = -y^{2} \frac{r_{te}}{r_{tt}} + 2zr_{te}$$
(A.27)

Finally, Equation A.27 can be rearranged to form the equation of a paraboloid:

$$\frac{x^2}{r_{te}} + \frac{y^2}{r_{tt}} = 2z \tag{A.28}$$

A.2 Vertical Pressure Distribution

The overall area of the contact patch can be approximated by dividing the normal force F_z by the inflation pressure P (Equation 2.1). From this, the height of tire deformation h can be determined by finding the z for which an ellipse defined by Equation A.28 gives the same area:

$$A = \pi \sqrt{2hr_{tt}} \sqrt{2hr_{te}} \tag{A.29}$$

$$\frac{F_z}{P} = 2\pi h \sqrt{r_{te} r_{tt}} \tag{A.30}$$

$$h = \frac{F_z}{2\pi P \sqrt{r_{te} r_{tt}}} \tag{A.31}$$

The vertical deformation profile $\delta_z(x, y)$ is the difference between the undeformed toroid z-position (determined by Equation A.28) and this height h, which is constrained to be non-negative:

$$\delta_z(x,y) = \max\{h - z(x,y), 0\}$$
 (A.32)

Using Equations A.28, A.31, and A.32, $\delta_z(x, y)$ can be solved as:

$$\delta_z(x,y) = \frac{F_z}{2\pi P \sqrt{r_{te} r_{tt}}} - \frac{x^2}{2r_{te}} - \frac{y^2}{2r_{tt}}$$
(A.33)

$$\delta_z(x,y) = \frac{F_z}{2\pi P \sqrt{r_{te} r_{tt}}} \left[1 - \left(\frac{x}{\sqrt{\frac{F_z}{\pi P}} \sqrt[4]{\frac{r_{te}}{r_{tt}}}}\right)^2 - \left(\frac{y}{\sqrt{\frac{F_z}{\pi P}} \sqrt[4]{\frac{r_{tt}}{r_{te}}}}\right)^2 \right] \quad (A.34)$$

It is assumed that the vertical pressure distribution $\sigma_z(x, y)$ is proportional to the vertical deformation $\delta_z(x, y)$ by the vertical stiffness k_z :

$$\sigma_z(x,y) = k_z \delta_z(x,y) \tag{A.35}$$

The integral of this force distribution over the area of the contact patch must be equal

to the normal force F_z :

$$F_z = \int_A \delta_z(x, y) dA \tag{A.36}$$

To simplify algebra, it's easiest to substitute x and y by normalized variables u and v:

$$u = \frac{x}{\sqrt{\frac{F_z}{\pi P}}\sqrt[4]{\frac{r_{te}}{r_{tt}}}} \tag{A.37}$$

$$v = \frac{x}{\sqrt{\frac{F_z}{\pi P} \sqrt[4]{\frac{T_{tt}}{\tau_{te}}}}}$$
(A.38)

Combining Equations A.34, A.35, A.36, A.37, and A.38, the vertical stiffness k_z can be solved as:

$$F_{z} = \int_{-1}^{1} \left(\sqrt{\frac{F_{z}}{\pi P}} \sqrt[4]{\frac{r_{te}}{r_{tt}}} dv \right) \int_{-\sqrt{1-v^{2}}}^{\sqrt{1-v^{2}}} \left(\sqrt{\frac{F_{z}}{\pi P}} \sqrt[4]{\frac{r_{tt}}{r_{te}}} du \right) k_{z} \delta_{z}(u,v) \quad (A.39)$$

$$F_z = \frac{F_z^2}{2\pi^2 P^2 \sqrt{r_{te} r_{tt}}} k_z \int_{-1}^{1} dv \int_{-\sqrt{1-v^2}}^{\sqrt{1-v^2}} du \left(1 - u^2 - v^2\right)$$
(A.40)

$$F_{z} = \frac{F_{z}^{2}}{2\pi^{2}P^{2}\sqrt{r_{te}r_{tt}}}k_{z}\int_{-1}^{1}dv \left[\frac{4}{3}\left(1-v^{2}\right)^{\frac{3}{2}}\right]$$
(A.41)

$$F_z = \frac{F_z^2}{2\pi^2 P^2 \sqrt{r_{te} r_{tt}}} k_z \left[\frac{\pi}{2}\right]$$
(A.42)

$$k_z = \frac{4\pi P^2 \sqrt{r_{te} r_{tt}}}{F_z} \tag{A.43}$$

From this, the vertical pressure distribution $\sigma_z(x, y)$ can be determined using Equations A.34 and A.35:

$$\sigma_z(x,y) = 2P \left[1 - \left(\frac{x}{\sqrt{\frac{F_z}{\pi P}} \sqrt[4]{\frac{T_{te}}{r_{tt}}}} \right)^2 - \left(\frac{y}{\sqrt{\frac{F_z}{\pi P}} \sqrt[4]{\frac{T_{tt}}{r_{te}}}} \right)^2 \right]$$
(A.44)

This is the final result - the vertical pressure distribution using a paraboloid approximation to a toroidal tire. Setting $\sigma_z(x, y) = 0$ yields the equation of the contact patch shape - an ellipse.

A.3 Comparison of Approximated and True Toroidal Geometries

The paraboloid approximation assumes that the vertical deformation h is small relative to the tire radii r_{te} and r_{tt} . Therefore, deviations between the approximated and true toroidal geometry are largest when the vertical deformation is highest. For the tire contact patches measured in Chapter 2, these are the most heavily-loaded cases: when $F_z = 3000$ N for the 180/55R17 tire and when $F_z = 4000$ N for the 300/35R18 tire.

Figure A.1 compares the force distributions for these two cases as predicted by the paraboloid approximation and the true toroidal geometry (Equation A.10). Each plot shows the level curves for the normal pressure $\sigma_z(x, y)$ at 0.00P - 1.75P in increments of 0.25P. Even for these two cases, the paraboloid is a close approximation of the true toroidal geometry - deviations are small.



Figure A.1: Level curves for normal pressure as predicted by elliptical approximation and true toroidal geometry, shown at normal pressures of 0.00P - 1.75P in increments of 0.25P, where P is the inflation pressure (top: 180/55R17 tire, bottom: 300/35R18 tire)

Appendix B

Suspension Design Software

The author developed a code in MATLAB called **suspmodel**. It is included as an electronic appendix of the dissertation. For a given set of suspension geometry parameters, it generates:

- positions of all joints
- kinematic mappings
- Jacobians
- suspension design criteria
- forces in outer ball joints
- lookup tables for use in real-time Simulink code that operates the prototype suspension system
- formatted plots of design criteria
- formatted 3D plots of suspension layout

At the heart of this code is an auto-generated MATLAB script from an Autolev program written by the author. It computes the joint positions and mappings for a given suspension position as well as the joint forces in the outer ball joints. The rest of the script automates this process for a range of suspension movement then generates suspension design criteria, plots, and lookup tables needed for real-time operation of the prototype suspension system.

The author has taken care to insert many comments into these scripts. So, while the following sections provide some insight into the basic structure and variable names of the code, most of the understanding of the scripts is found by reading through them.

B.1 Structure

The code is run by opening the main script, suspmodel_main. From this script, one can observe the basic structure of the code. The first set of scripts are needed to set suspension geometry and calculate and plot the design criteria:

- suspmodel_constants_v9_mc300, suspmodel_constants_v9_mc180, and suspmodel_constants_v9_car255 set the parameters that describe the suspension geometry (currently at version 9) for three different types of tires: 300/35R18 motorcycle tires, 180/55R17 motorcycle tires, and 255/40R18 car tires, respectively. The prototype suspension is primarily designed for use with 300/35R18 motorcycle tires. The other two configurations are used to allow tire testing on additional types of tires. Only one should be run for each simulation run.
- suspmodel_configsolver sets variables that are used to configure the solver, which includes specifying the camber, steer, and vertical suspension positions used for calculation.
- suspmodel_runsolver runs the main solver script (suspmodel_solver_v3n, which is currently at version 3n) for each position and assigns its outputs to variable names that describe the positions of all joints and the mappings.
- **suspmodel_postproc** generates suspension design criteria from the solver output data.
- suspmodel_parameterplots plots the suspension design criteria.

• suspmodel_plot plots the suspension layout in 3D.

The next set of scripts are needed to calculate additional parameters and format lookup tables for use with the real-time Simulink code that operates the prototype suspension system, and may be skipped if only the suspension design criteria are needed:

- suspmodel_constants_addl_v1 sets additional geometry parameters that are *not* used in the main solver, but are used to determine mappings for load cells and the suspension spring. These parameters do not depend on tire choice, unlike the main geometry parameter script.
- suspmodel_lcacalcs calculates the reaction forces on the inboard side of the lower control arm (LCA). The main suspension solver considers only a kinematic skeleton where the inboard side of the LCA is represented by a revolute joint, but in actual construction it is a pair of ball joints. This calculations are necessary since one load cell is located on one of these ball joints.
- **suspmodel_loadcellcalcs** calculates the forces in the suspension load cells from the main solver output data.
- suspmodel_postproc_simulink post-processes the mappings into lookup tables formatted for use with the real-time Simulink code that operates the prototype suspension system.
- suspmodel_fix_zeropos resets the zero positions of the lookup tables to avoid suspension recalibration when only a simple tire change is performed. This is done by assuming that one geometry set (namely the set that specifies the 300/35R18 motorcycle tires).
- **suspmodel_runinfo** generates a variable that accompanies the lookup tables and describes the simulation run.

There are also a few lines that specify whether or not to save the plots or lookup tables in files:

- saveparamplots is the flag that determines whether or not the suspension design criteria plots are saved. 0 is no; 1 is yes.
- save3dplots is the flag that determines whether or not the 3D suspension layout plots are saved. 0 is no; 1 is yes.
- save KinematicsXXX_all.mat saves all of the variables. Choose XXX to describe the specific simulation run.
- save KinematicsXXX.mat FK RK SuspConfig saves only the variables that are needed to operate the Simulink code for the prototype suspension system. Choose XXX to describe the specific simulation run.

B.2 Measurement Origin and Naming Conventions

The origin used for absolute measurements is the point on the ground directly below the vehicle CG. The axes used are the vehicle axes as described in Section 4.2.4.

The naming of variables is systematic. Here are the main types:

- Variables that start with x, y, or z denote the absolute positions of a point from the origin along the vehicle axes. For example, xLOBJ yLOBJ zLOBJ denote the x, y, and z positions of *LOBJ*.
- Variables that start with dx, dy, or dz denote offsets in position from some other point specified in axes that may or may not be the vehicle axes. For example, dx0TRJ dy0TRJ dz0TRJ denote the x, y, and z offsets of OTRJ from LOBJ in the axes of K. The details of each offset are given in comments where they are used.
- Variables that start with q denote orientation angles of members. If there is more than one angle for a given member, the variable name ends with the appropriate axis. For example, qLCA denotes the angular orientation of LCA and qTRx qTRz denote the angular orientations of TR about x and z axes.

- Variables that start with l are lengths. These may be lengths of members (e.g. lLCA is the length of LCA) or of design criteria (e.g. l_va is the vertical suspension actuator effective moment arm length l_{va}).
- Variables that start with h are heights. These may be heights of geometry (e.g. hVCG is the vehicle CG height) or of design criteria (e.g. h_rc is the roll center height h_{rc}).
- Variables that start with r are radii. For example, rSMA is the radius of SMA.
- Variables that start with Fx, Fy, or Fz are forces and those that start with Mx, My, or Mz are moments. They are generated in response to a unit amount of tire force or moment at the contact point *CP*. All of these are measured along the vehicle axes. For example, FxV is the resulting longitudinal force at the vehicle CG from tire forces and MxCAJ is the x-axis torque exerted at *CAJ* by the camber actuator.
- Variables that start with D are deviations (similar to Δ). For example, D_dx is the longitudinal tire scrub, which is the deviation of d_x .
- The suspension position variables are h, gamma, and delta. The range of these variables used in calculation are h0s, gamma0s, and delta0s.

B.3 Suspension Geometry Parameter Variables

The scripts suspmodel_constants_v9_XXX and suspmodel_constants_addl_v1 contain all of the geometry parameters of the suspension (where XXX represents the specific tire choice). These are the parameters that the designer will be changing to iterate through different suspension system designs. It is recommended that separate copies of these files should be generated for different designs. The specific ones used for a simulation can be assigned in suspmodel_main.

In this dissertation, the design given as the current design and for which the design criteria in Section 5.2 were plotted is given by the geometric parameters in the default suspmodel_constants_v9_mc300 script.

Here is a list of the variables used in suspmodel_constants_v9_XXX, the main parameter script. They are given in the same order as the steps in Section 5.1. The additional parameters in suspmodel_constants_addl_v1 are not described here, but are well-commented in the script itself.

Vehicle and Tire Parameters

- hVCG is the vehicle CG height above ground at the nominal suspension position.
- 1V is the longitudinal offset from the vehicle CG to to the contact point *CP* at the nominal suspension position. This should be positive for the front axle and negative for the rear.
- rT is the same as r_{tt} and rTO is the same as r_{te} in this dissertation. They are defined in Section 4.2.3.

Outer Ball Joint Parameters

- qC is the caster angle and dC is the caster offset. The offset is the longitudinal distance from the wheel center WC and the steer axis. If it is zero, then the steer axis passes through the wheel center WC in a view from the x z plane.
- 1K is the length of the knuckle K as measured between LOBJ and UOBJ in the x-z plane. hK is the height of LOBJ above its "ideal" location, which is on the circle defining the center of the tire tread. This is measured along the knuckle axes, which have the z-axis going through LOBJ and UOBJ (as projected into the xz-plane) and the y-axis pointing along the direction of rotation of the wheel.
- dKL dKU are the lateral offsets from the LOBJ and UOBJ, respectively, and the x - z plane that passes through the wheel center WC. When they are zero, there is no kingpin angle and no scrub radius; the z-axis of the knuckle passes through LOBJ and UOBJ. When they are nonzero but the same, there is no kingpin angle but there is a scrub radius; the z-axis of the knuckle still passes through both outer ball joints. Then the two parameters are dissimilar, there

is a nonzero kingpin angle and the z-axis of the knuckle does not pass through UOBJ.

Control Arm Parameters

- yLIBJ zLIBJ define the absolute position of LIBJ. The x position is determined by other parameters.
- 1LCA 1UCA are the lengths of LCA and UCA, respectively.
- qLCAz is the z-axis rotation of the inboard axis of the lower control arm LCA from the longitudinal axis. When this is zero, the inboard LCA axis is exactly longitudinal.

Camber Moment Arm Parameters

- yCAJ zCAJ define the absolute position of CAJ. The x position is determined by other parameters.
- 1CMA is the length of CMA.

Vertical Suspension Linkage Parameters

- dxVAJ yVAJ zVAJ define the absolute position of VAJ. The offset in x is measured relative to the wheel center WC.
- dxLPRJ dyLPRJ dzLPRJ define the position of *LPRJ* on **LCA**. These are measured in **LCA** coordinates, which has the y-axis going through *LOBJ* and *LIBJ* and the x-axis pointing forward.
- 1PR rVMA are the length of PR and the radius of VMA.

Outer Tierod Joint Parameters

dxOTRJ dyOTRJ dzOTRJ define the offset from LOBJ to OTRJ on the knuckle
 (K) in K coordinates. As mentioned earlier, these knuckle axes have the z-axis going through LOBJ and UOBJ and the y-axis pointing along the direction of rotation of the wheel.

Steering Linkage Parameters

- dxSAJ ySAJ zSAJ define the absolute position of SAJ. The offset in the x direction is relative to the wheel center WC.
- 1TR rSMA are the length of TR and the radius of SMA, respectively.

B.4 Solver Output Variables and Mappings

The outputs of the solver are given as 4D arrays. Each entry is the value of the variable for a particular suspension position (q_t) with a particular effort input at the tire (e_t) . The indices of these arrays are as follows:

- 1. Vertical suspension position (h). These correspond to the positions given in the variable h0s.
- 2. Camber angle (γ) . These correspond to the positions given in the variable gamma0s.
- 3. Steer angle (δ). These correspond to the positions given in the variable delta0s.
- 4. Effort input number. There are six of these one for each tire force and moment, in order of F_{xt} , F_{yt} , F_{zt} , M_{xt} , M_{yt} , and M_{zt} . For each input number, the value of the corresponding effort input is set to one and all of the others to zero.

The solver is run once for each entry in these 4D arrays. For example, for the entry with indices (1,3,2,5), the solver is run with the first vertical position in h0s, the third camber angle in gamma0s, the second steer angle in delta0s, and the fifth input number (which is when all $e_t = 0$ except for the fifth one, which means that $M_{yt} = 1$).

The kinematic mappings are found by looking at the values found in the 4D configuration variables (e.g. qSMA is the angle of the steering moment arm for each configuration and effort input, which is the same as the steering actuator angle). The Jacobians are found by looking at the 4D effort variables (e.g. MxSAJ is the torque at SAJ, which is the same as the steering actuator torque).

The complete list of solver output variables is given in the variable **outvars**, which is defined in **suspmodel_configsolver**.

Note that because configuration is independent of effort, the configuration variables have identical values for each effort input number. For example, indices (1,3,2,1), (1,3,2,4), and (1,3,2,6) are all the same for configuration variables such as qSMA.

B.5 Design Criteria

The design criteria are generated from the variables described in Section B.4 by the script suspmodel_postproc. They are given as 3D arrays. The three indices are the same as the first three indices given above for the solver output variables - these describe the suspension position. The specific definition and meaning of each can be found in the suspmodel_postproc script.

B.6 Viewing and Saving Plots

Each plot can be saved to an eps or pdf file. To change the way this saving is performed, edit the script **saveplot**. By default, this is configured to generate eps files, which can be done directly from MATLAB. In addition, if one has MikTex installed, epstopdf can be used to convert this to a pdf. By default, this option is disabled so that it is compatible with all users that have only MATLAB.

To set whether or not the saveplot script is called to save the plots, set the saveparamplots and save3dplots parameters in suspmodel_main; 1 saves the plots and 0 omits saving.

Note that the 3D plot of the suspension layout is a true 3D plot in MATLAB. Therefore, it can be rotated and stretched to the user's liking using MATLAB's standard 3D plotting tools. This can help dramatically with understanding how these points are laid out in 3D.

B.7 Interfacing With Real-Time Suspension Code

The real-time Simulink code for the prototype suspension system uses lookup tables to perform all of its kinematic and Jacobian mappings. These are generated as the structures FK and RK. FK represents the forward kinematics and uses joint angles as its inputs; RK represents the inverse (or reverse) kinematics and uses tire positions as its inputs. The SuspConfig variable contains information about the simulation run that are saved along with the two lookup tables.

Included in the code provided are six mat files. They are the outputs using each of the three tire sizes. Three files can be used directly in the Simulink code and the other three have saved all of the simulation outputs. Although useful for suspension analysis, these files are too large for use with the real-time code. These files are:

- Kinematics300 is used with the real-time suspension code and a 300/35R18 motorcycle tire. This is used for the Avon and Metzeler tires used in the dissertation.
- Kinematics300_all is the same case as above, but has all simulation variables saved.
- Kinematics180 is used with the real-time suspension code and a 180/55R17 motorcycle tire. This is used for the Dunlop tire used in the dissertation.
- Kinematics180_all is the same case as above, but has all simulation variables saved.
- Kinematics255car is used with the real-time suspension code and a 255/40R18 car tire. The car tire was not used for this dissertation, but has enabled colleagues to use the suspension testbed to characterize other tires for additional research.
- Kinematics255car_all is the same case as above, but has all simulation variables saved.

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