Late Phasing Homogeneous Charge Compression Ignition
Cycle-to-Cycle Combustion Timing Control With Fuel Quantity Input

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Abstract—Late-phasing homogeneous charge compression ignition (HCCI) operating conditions have the potential to expand the useful operating range of HCCI. However, these conditions exhibit significant variation in combustion timing and work output from one cycle to the next. Cyclic variations in the combustion timing of HCCI combustion at late-phasing operating conditions can be removed through the use of cycle-to-cycle control of fuel injection quantity. A nonlinear, discrete-time model of the recompression HCCI process captures the oscillations in late-phasing HCCI; when this model is linearized, it represents these dynamics as a pole on the negative real-axis. A simple lag compensator eliminates the oscillations in combustion phasing and drastically improves the operability of late-phasing HCCI in both simulation and experiment.

I. INTRODUCTION

Homogeneous charge compression ignition (HCCI) internal combustion engines provide a promising technology relative to conventional spark-ignition (SI) and compression-ignition (diesel) engines [1], [2]. HCCI engines are more efficient and emit less \( NO_x \) than SI engines, and they emit less \( NO_x \) and particulate matter than diesel engines [3], [4]. One of the challenges facing their further implementation is the limited load range in which they operate. Yoshizawa et al. [5] illustrated that one potential method for increasing the load range of HCCI engines is to operate at late combustion phasings, which can be achieved in recompression HCCI by retaining smaller amounts of exhaust in the cylinder [6], [7]. However, Wagner et al. [8] showed that cyclic variations in indicated mean effective pressure (IMEP) grow significantly as the retained exhaust ratio is reduced, and Shahbakhti et al. [9] found that cyclic variation in the start of combustion timing increased under similar conditions.

Fig. 1 shows that the combustion phasing \( \theta_{50} \), the crank angle at which 50% of the mass of the fuel in the cylinder is burned, also exhibits significant variation from one cycle to the next as the retained exhaust ratio is reduced. The data shown was obtained by operating an experimental engine at two different conditions, each with the same engine speed, fueling rate, fuel injection timing, and intake valve timing. The only input which differs between the two conditions is the exhaust valve timing. In the nominal case, shown prior to cycle 50, small cycle-to-cycle variations exist in the combustion timing. After the exhaust valve timing is shifted later, that variation increases significantly. These cyclic variations in late-phasing HCCI can be reduced by controlling the exhaust valve timing on a cycle-by-cycle basis [6]. That approach achieves good results, but its implementation requires that the exhaust valve timing on each cylinder be independently controllable. Instead, using fuel injection quantity as an input has the advantage of being easily individually controllable in each cylinder, making it a more production-oriented technology.

The primary challenge surrounding the use of fuel injection quantity as a suitable input is its non-minimum phase nature, which is caused by charge cooling. The degree of charge cooling that takes place in the cylinder affects the combustion phasing by changing the temperature of the contents of the cylinder prior to combustion on the following cycle. Unfortunately, the extent of charge cooling is difficult to capture in a simple, control-oriented model. Thus, any controller seeking to use fuel quantity inputs to control combustion phasing must be robust to different assumptions about the extent of charge cooling that takes place in the cylinder as a result of changes in the fuel injection quantity.

In the next section, a brief description of the physical model shows how it incorporates fuel injection quantity as an input to control combustion timing. The model is linearized,
and it illustrates that the oscillations in combustion timing at late-phasing conditions are driven by a pole on the negative real axis. Two different assumptions about the degree of charge cooling result in different zero locations when analyzed on root loci. However, a single controller successfully moves the negative-axis pole onto the positive axis in models based on both assumptions. Finally, that controller drastically reduces the oscillations in combustion timing and IMEP on an experimental engine.

II. MODEL DESCRIPTION

Ravi et al. [10] originally presented the nonlinear model upon which this work is based. The model is a discrete-time, nonlinear model of the HCCI combustion process that features three states, a single-input, and a single output. The model is of a single cylinder of Stanford’s 4-cylinder HCCI engine and simulates the engine operating a recompression HCCI strategy that utilizes direct-injected gasoline and air. When the value of the integral equals the Arrhenius threshold, combustion begins. The Arrhenius integral is calculated from the state angle to the start of combustion as in Eq. (1).

\[
\int R R d\theta = \int_{\theta_S}^{\theta_S + \Delta \theta} \exp \left( \frac{E_a}{R T} \right) \frac{[f]}{[O_2]}^b d\theta = K_{th} \tag{1}
\]

In Eq. (1), \(E_a\) is the activation energy required for the fuel, \(A_{th}\) is a pre-exponential factor, \(R_s\) is the universal gas constant, \(T\) is the temperature, \([f]\) is the fuel concentration, and \([O_2]\) is the oxygen concentration in the cylinder.

The physical nonlinear model can be written as a state-update equation and output equation that illustrate how the system dynamics evolve on a cycle-to-cycle basis.

\[
x_{k+1} = f \left( [O_2] \right)_{k+1}, T(\theta_{k}), y_k = h \left( [O_2] \right)_{k+1}, T(\theta_{k}), [f](\theta_{k}) \tag{2}
\]

A. Model Structure

Fig. 2 illustrates the model states, input, and output on an in-cylinder pressure trace.

1) Model States: The model uses the following three states:

\[
x = \left[ [O_2] \right), T(\theta_{k}), [f] \left( \theta_{k} \right) \right] T
\]

where \([O_2] \left( \theta_{k} \right)\) is the oxygen concentration at the state angle, \(T(\theta_{k})\) is the in-cylinder mixture temperature at the state angle, and \([f] \left( \theta_{k} \right)\) is the fuel concentration at the state angle. The model state angle, \(\theta_{k}\), is chosen to be \(\theta_{k} = 660\) CADaTDc (Crank Angle Degrees after Top Dead Center combustion) because it occurs after the input and prior to the output, resulting in a model formulation that does not contain a feedthrough matrix.

2) Model Input: The model uses the following input:

\[
u = n_f
\]

where \(n_f\) is the number of moles of fuel injected into the system. The model assumes that the fuel is injected at the end of the recompression stage of the engine cycle.

3) Model Output: The model uses the following output:

\[
y = \theta_{50}
\]

where \(\theta_{50}\) is the combustion timing. The angle of 50% fuel mass burned is the metric for combustion timing instead of the location of peak pressure, LPP, or the start of combustion angle, \(\theta_{SOC}\), because LPP can sometimes occur prior to combustion for cycles with very late combustion events, while \(\theta_{SOC}\) can be similar for many combustion events that have different durations [11], [12].

B. Combustion Phasing Determination

The model determines \(\theta_{SOC}\) by integrating a global Arrhenius reaction rate equation for the combustion of gasoline and air. When the value of the integral equals the Arrhenius threshold, combustion begins. The Arrhenius integral is calculated from the state angle to the start of combustion as in Eq. (1).

\[
\int R R d\theta = \int_{\theta_S}^{\theta_S + \Delta \theta} \exp \left( \frac{E_a}{R T} \right) \frac{[f]}{[O_2]}^b d\theta = K_{th} \tag{1}
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\]

C. Time Domain Performance of Model

Fig. 3 shows the response of the nonlinear model to a step change in exhaust valve timing. The late-phasing HCCI condition exhibits cycle-to-cycle oscillations, indicating that the model’s dynamics are representative of the dynamics on the engine. The oscillations die away after several cycles in the model whereas they do not on the experimental engine, because the model treats certain physical phenomena as noise, such as variations in airflow from one cycle to the next.

III. ANALYSIS OF THE FUEL QUANTITY INPUT

The nonlinear model captures the dynamics of HCCI combustion quite well, but does not present the clearest path to control development. Instead, linearizing the nonlinear model leads quickly to controllers that can then be implemented on the engine.
A. Linearized Model

A linearization of the nonlinear model at a late operating condition with \( \theta_{50} = 12.9 \text{ CADaTDCc} \) yields the state-space system shown by the system of equations in Eq. 3.

\[
A = \begin{bmatrix} 0.44 & -0.053 & -0.15 \\ -0.014 & -0.26 & 0.051 \\ 0 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0.031 \\ -0.022 \\ 1 \end{bmatrix}^T, \quad C = \begin{bmatrix} -0.061 \\ -1.70 \\ 0.013 \end{bmatrix}
\]  

(3)

Here, the state, \( \hat{z} = [(O_2), T, [f]]^T \), is the oxygen concentration, temperature, and fuel concentration of the system normalized about the steady-state state values corresponding to the late operating condition. The input, \( \hat{u} = n_f \), is the number of moles of fuel input into the system normalized about the late operating condition. Finally, the output, \( \hat{y} = \theta_{50} \), is the normalized combustion phasing corresponding to the late operating condition.

Furthermore, the state-space system can be combined into a transfer function that shows the pole and zero locations of the linearized system.

\[
G(z) = 0.048 \frac{(z-0.49)(z-1.51)}{z(z-0.44)(z+0.26)}
\]

(4)

The transfer function mapping the fuel quantity input to the combustion timing output has two zeros and three poles. One of the poles lies on the negative real-axis, which leads to oscillations on every-other cycle in the system output. This open-loop pole location explains how the model is able to capture the oscillatory dynamics of the late phasing condition.

Additionally, the transfer function has a negative DC gain, meaning that if there is a step increase in the quantity of fuel injected into the cylinder, then the combustion phasing will be earlier after the system dynamics settle. The negative DC gain aligns well with the physical intuition about the problem: by adding more fuel to the system, more energy is released during combustion, resulting in hotter retained exhaust and a higher state temperature on the subsequent cycle. This higher state temperature leads to earlier combustion timings.

Finally, one of the zeros is a non-minimum phase zero, located at \( z = 1.51 \), to the right of \( z = 1 \). The non-minimum phase zero causes the system’s response first to move in the opposite direction of the steady-state response on the first cycle and then to move in the direction of the steady-state response on subsequent cycles. This means that a step increase in fuel quantity on cycle \( k \) will first lead to a later combustion phasing on cycle \( k + 1 \) before leading to earlier phasings on cycles \( k + 2 \) and beyond. The non-minimum phase behavior results from additional charge cooling of the retained exhaust, which occurs due to the increased fuel quantity injected in the cylinder. The charge cooling on cycle \( k \) results in a lower state temperature and a later combustion phasing on cycle \( k + 1 \).

Fig. 4 shows the response of the linearized model to a 10% step increase in fuel quantity. The non-minimum phase behavior, negative DC gain, and oscillatory behavior are all visible in the output’s response.

B. Examination of Charge Cooling

Roelle et al. [13] illustrated the non-minimum phase relationship between fuel quantity and combustion phasing. However, that model showed a smaller impact of charge cooling on the combustion phasing on cycle \( k + 1 \) for a step change in fuel quantity on cycle \( k \) than this model shows.

Fig. 5 shows experimental results of charge cooling from the engine. Asterisk symbols show the effect of main fuel mass injected on cycle \( k \) against the combustion phasing from cycle \( k + 1 \), the immediately subsequent event. Also, a dashed trend line shows a linear fit of that data. As the model predicts, the line slopes positively, indicating that as
fuel quantity increases, combustion phasing retards on the
following cycle. Puls-sing symbols illustrate the effect of
fuel mass on cycle k versus the combustion phasing from
cycle k + 2. Here, a solid trend line shows the linear fit of
the data. The solid line slopes negatively, indicating that as
fuel quantity increases, combustion phasing advances on
the second cycle. The slope of the solid line is -1.91, steeper
than the slope of the dashed line, 0.45. The relative slopes
of the lines indicate some level of disagreement between the
model and experimental data. However, it is not clear how
much impact this disagreement would have on the ability
of fuel quantity to reduce cyclic variation in the combustion
phasing of late-phasing HCCI.

One simple strategy to test the impact charge cooling has
on fuel quantity as an input is to simply modify the linear
model to reduce the non-minimum phase characteristic of
the model, and then design one controller that moves the
system poles to desirable locations for both models. This is
done by modifying the input matrix, B, in Eq. 3 so that the
effect of changing the amount of fuel injected on cycle k will
have less impact on the phasing on cycle k + 1. By setting
B(2) = 0, any changes in fuel quantity will not affect state
temperature on the following cycle through the input; those
changes will instead only effect state temperature through
the fuel and oxygen states in the system dynamics. Thus,
setting B(2) = 0 establishes a lower bound on the amount
of charge cooling in the system.

Fig. 6 shows the response of the linearized model and
the modified linearized model to an identical step input. The
only difference between the two results from the differ-
ing input matrices, which can be seen between cycles 1 and 2.
The two systems follow parallel trajectories after the cycle
2 because they have the identical system matrix, A.

Eq. 5 shows the transfer function for the modified lin-
earized system. Both the unmodified and modified systems
have poles at \( z = 0 \), \( z = 0.44 \), and \( z = -0.26 \), and the
unmodified system has a zero at \( z = 0.49 \) while the modified
system has a zero at \( z = 0.48 \). The key difference between
the two systems is in the non-minimum phase zero: in the
unmodified system, the zero is located at \( z = 1.51 \), while
in the modified system, the zero is located at \( z = 6.57 \).
Thus, the different assumptions about charge cooling can be
described simply as different zero locations.

\[
G_{mod}(z) = \frac{0.011(z - 0.48)(z - 6.57)}{(z - 0.44)(z + 0.20)}
\]  

Fig. 6. Comparison of step responses of original and modified models

The two different zero locations can be visualized in
Figs. 7 and 8, which show the root loci for both systems.

IV. CONTROL DESIGN

The controller for the late-phasing system needs to fulfill
two goals: first, eliminate the cycle-to-cycle oscillations in
\( \theta_{50} \), and second, be robust to model uncertainty regarding
charge cooling. A lag compensator with a negative gain,
shown in Eq. 6, meets both of these objectives.

\[
K_{late}(z) = -9 \frac{z}{z - 0.7}
\]  

The intuition behind the compensator is straightforward.
The negative system gain moves the negative real axis pole
back into the right-half plane in both discrete-time systems,
removing the oscillating dynamics. The zero in the lag
compensator is placed at the origin, canceling the open-loop
pole at \( z = 0 \) in both systems. The compensator adds a pole
at \( z = 0.7 \) to both systems, effectively filtering oscillations
out of the system. The compensator accomplishes both its
objectives while keeping all the closed-loop poles inside the
unit circle and therefore not inducing any stability concerns.

Fig. 9 shows a root locus illustrating the control design
for the unmodified model, and Fig. 10 shows a root locus
for the modified model. The figures illustrate that the closed-
loop poles for both systems lie at similar locations. In the
unmodified system, the closed-loop pole locations occur at
\( z = 0.51 \) and \( z = 0.40 \pm 0.56i \), while in the modified

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Fig. 5. Experimental results showing the effect of fuel mass injected on
cycle \( k \) on combustion phasing on cycle \( k + 1 \) and combustion phasing on
cycle \( k + 2 \)
system, the closed-loop pole locations occur at $z = 0.49$ and $z = 0.25 \pm 0.66i$.

V. RESULTS

Experimental tests validating the controller’s performance were performed on the Stanford multi-cylinder HCCI engine, which is pictured in Fig. 11. The engine is a 2.2L General Motors 4-cylinder ECOTEC engine equipped with a variable valve actuation system, Bosch direct fuel injection system, and in-cylinder pressure transducers. The experimental conditions are shown in Tab. 1.

<table>
<thead>
<tr>
<th>Condition</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Engine speed</td>
<td>1800 RPM</td>
</tr>
<tr>
<td>Intake valve opening</td>
<td>430 CADaTDCc</td>
</tr>
<tr>
<td>Intake valve closing</td>
<td>570 CADaTDCc</td>
</tr>
<tr>
<td>Exhaust valve opening $\theta_{EV C} - 140$</td>
<td></td>
</tr>
<tr>
<td>End-of-injection timing</td>
<td>420 CADaTDCc</td>
</tr>
</tbody>
</table>

Fig. 12 shows experimental results from cylinder 1 of the research engine. There are three different operating conditions shown in the test. The first condition has a nominal exhaust valve timing in both cylinders and a constant fuel injection mass. In the second condition, the exhaust valve timing moves later at cycle 146, and significant oscillations appear due to the dynamics at late-phasing conditions. Finally, the controller reduces the oscillations of late-phasing HCCI by changing the fuel mass on a cycle-to-cycle basis.

The controller successfully reduces the magnitude of the oscillations in $\theta_{50}$. The controller is switched on at cycle 300; the reduction in the peak-to-peak oscillations is apparent. The controller reduces the coefficient of variation of $\theta_{50}$ from 0.82 to 0.24. The other three cylinders showed reduced coefficient of variation values in closed-loop operation as well. The controller also manages to reduce the cycle-to-cycle variations in IMEP, even though that is not an explicit goal of the controller, from 0.13 to 0.062.

VI. FUTURE WORK

The controller utilizes $\pm 1$ mg fuel quantity changes to achieve these results, which is significant. However, Figs. 9
and 10 illustrate the controller gain could be reduced while keeping the closed-loop pole locations in the right-half plane. Reducing the gain would decrease actuator effort, and would lead to similar improvements. Additionally, the models for using main injection fuel mass to control cyclic variations could be added to model predictive controllers that operate throughout the entire HCCI combustion phasing range [14].

VII. CONCLUSIONS

The performance of late-phasing homogeneous charge compression ignition operating conditions can be improved through the use of cycle-to-cycle fuel injection quantity control. A nonlinear model captures the oscillations that are present in experimental results and, when linearized, represents these oscillations as a pole on the negative real axis. A simple lag compensator, based on output feedback, eliminates the cyclic variations in combustion timing despite uncertainties about the extent of charge cooling due to changes in the quantity of fuel injected.

REFERENCES


