PERFORMANCE GUARANTEES FOR HAZARD BASED LATERAL VEHICLE CONTROL

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ABSTRACT

Today's vehicles are incorporating many advanced driver assistance systems and in the near future it will be likely to have increased capabilities such as lanekeeping assist systems. These systems will be an integral part of the driving experience, aiding the driver in avoiding hazardous obstacles. One approach for these systems is to represent the hazards as artificial potential fields that add control inputs to move the vehicle towards safe regions on the road. This paper focuses on bounding the lateral motion of a vehicle for a lanekeeping system. A Lyapunov approach is used where the bounding function consists of the artificial potential energy associated with the controller, the kinetic energy in the lateral and yaw modes, and energy terms that are dependent on vehicle heading. In order to achieve this bound, a condition has to be met for the lookahead distance and the location of the control force (which can also be interpreted as a condition on the decoupling of lateral and yaw modes). Using this bound, a potential field gain can be chosen to guarantee collision avoidance with fixed lateral obstacles.

NOMENCLATURE

U_x Longitudinal velocity (Body fixed frame)
U_y Lateral velocity (Body fixed frame)
r Yaw rate
m Mass
I_z Moment of inertia
a Distance from front axle to c.g.
b Distance from rear axle to c.g.
d Track width
C_f Front cornering stiffness
C_r Rear cornering stiffness
M Mass matrix
$\dot{q}$ Vector of velocity states
u_c Vector of control inputs
$f(\dot{q})$ Drift terms
$g(\dot{q},u_c)$ Controlled terms
F_{xrf}, F_{yrf} Longitudinal and lateral force on right front wheel
F_{xlf}, F_{ylf} Longitudinal and lateral force on left front wheel
F_{xrr}, F_{yrr} Longitudinal and lateral force on right rear wheel
F_{xlr}, F_{ylr} Longitudinal and lateral force on left rear wheel
ΔF_{xr} Differential rear tire force
ΔF_{xf} Differential front tire force
α_f Front cornering stiffness
α_r Rear cornering stiffness
w Vector of global position coordinates
ψ Heading angle (Global frame)
s Distance down roadway (Global frame)
e Lateral position of c.g. (Global frame)
e_{cf} Lateral position at control force location
e_{la} Lateral position at projected distance
x_{cf} Distance from c.g. to control force location
x_{la} Lookahead distance from control force location
V Potential function
k Potential function gain
z Lyapunov function variables
L Lyapunov function

1 Introduction

Today, many vehicles are being equipped with driver assistance systems such as anti-lock braking systems (ABS) and stability control. These systems are designed to aid the driver by preventing any unstable or unpredictable vehicle behavior. Although they provide stable dynamics, these control systems do not prevent the vehicle from avoiding hazardous environmental obstacles. Gerdes and Rossetter (2001) proposed a method for incorporating such links to the environment within the paradigm of artificial potential fields. In this work, a virtual control force derived from the potential is used to aid the driver in avoiding environmental obstacles. The control uses a combination of steering and differential braking to create the effect of a force applied at a point on the vehicle (Rossetter and Gerdes 2002). The potential function is an intuitive approach to representing levels of hazard. The potential's peaks correspond to large hazards while safe regions of the road are represented by low or zero potential function values. When the vehicle moves towards dangerous obstacles the kinetic energy of the vehicle is transferred to artificial potential energy as the control forces derived from the potential move the vehicle towards safer regions of the road. Since this control scheme is designed for driver assistance it does not cancel or alter the handling characteristics of the vehicle. As a result, the vehicle dynamics presented to the driver remain consistent and predictable.

This control approach is closely related to previous work in robotics (e.g. Khatib (1986) and Hogan (1985)) with the objective being a nominally safe driving environment as opposed to end-effector placement or trajectory generation. Similar ideas have been proposed by Reichardt and Schick (1994) who represented an autonomous vehicle as an electron and treated hazards as negative charges repelling the vehicle. Another similar concept by Hennessey *et al.* (1995) was the design of a 'virtual bumper' for collision avoidance. This concept attached imaginary springs and dampers around the vehicle. When an obstacle entered the virtual bumper space, the vehicle was controlled away from the hazard.

Although the potential field concept can be used for many types of hazards, it is an ideal framework for lanekeeping assist systems. In lanekeeping, the edges of the road represent hazards while the lane center, being the desired location, corresponds to the minimum of the potential. Unlike tracking controllers, the dynamics of the system are influenced by the combination of physical tire forces and the virtual control forces derived from a potential. These dynamics not only determine the stability from the driver's perspective, but also specify the vehicle's response to environmental obstacles. Previous work (Rossetter and Gerdes 2002) shows that there are two important results for stability. The first result is that the control force from the potential must be applied in front of the neutral steer point of the vehicle. This ensures that the vehicle will rotate away from the obstacle. The second stability result is the well known concept of lookahead, which in this case is accomplished by a projection into the lanekeeping potential. Although stability is crucial for a lanekeeping system, the path of the vehicle is also extremely important.

Unlike tracking controllers, this impedence approach to lanekeeping creates a system that does not track a desired trajectory. Although tracking controllers are ideal for autonomous systems, the ability to track a desired path with zero error is not important and, in fact, undesirable for driver assistance systems. Although exact tracking is not necessary, it is imperative to know locations on the road that are achievable by the vehicle. For lanekeeping, the vehicle can move anywhere within the lane, but it must not cross the lane boundaries. Therefore, the potential function must be large enough to prevent a lane departure. This paper presents a Lyapunov function that bounds the energy in the lateral and yaw directions without making small angle approximations on the vehicle heading. In order to obtain this bound, conditions must be satisfied for the lookahead distance and the application point of the control force. The appearance of these conditions in the bounding function is not surprising since they are needed for lateral stability. The Lyapunov function contains the lateral artificial potential energy, the lateral and yaw kinetic energy, and terms that are dependent on the vehicle heading. This Lyapunov function gives realistic performance bounds for the lateral motion of the controlled vehicle at a fixed longitudinal speed. This energy bound can be used to choose the potential field gain in order to avoid collisions with fixed lateral obstacles.

2 Vehicle Dynamics

The vehicle model used in the analysis is a simple three degree of freedom yaw plane representation with differential braking, shown in Figure 1. The equations of motion are

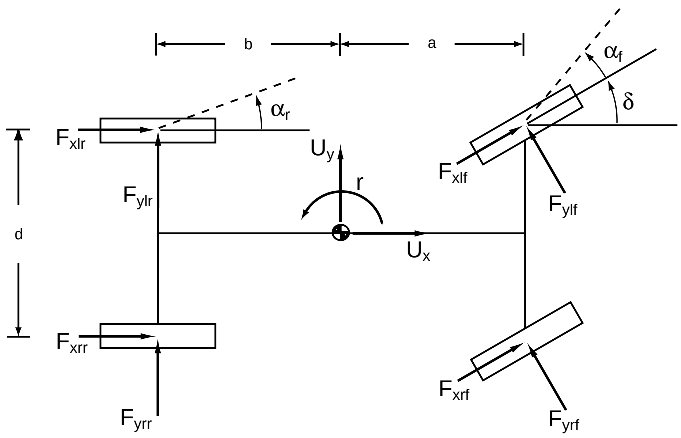

Figure 1. Vehicle Model

$$m\dot{U}_x = F_{xr} + F_{xf}\cos\delta - F_{yf}\sin\delta + mrU_y \qquad (1)$$

$$m\dot{U}_y = F_{yr} + F_{xf}\sin\delta + F_{yf}\cos\delta - mrU_x \quad (2)$$

$$I_z\dot{r} = aF_{xf}\sin\delta + aF_{yf}\cos\delta - bF_{yr} + \frac{d}{2}(\Delta F_{xr} + \Delta F_{xf}\cos\delta) \quad (3)$$

where

$$F_{xf} = F_{xrf} + F_{xlf} \quad (4)$$

$$F_{xr} = F_{xrr} + F_{xlr} \quad (5)$$

$$F_{yf} = F_{yrf} + F_{ylf} \quad (6)$$

$$F_{yr} = F_{yrr} + F_{ylr} \quad (7)$$

$$\Delta F_{xf} = F_{xrf} - F_{xlf} \quad (8)$$

$$\Delta F_{xr} = F_{xrr} - F_{xlr} \quad (9)$$

Assuming equal slip angles on the left and right wheels, the front and rear slip angles are

$$\alpha_f = \tan^{-1}\left(\frac{U_y + ra}{U_x}\right) - \delta \quad (10)$$

$$\alpha_r = \tan^{-1}\left(\frac{U_y - rb}{U_x}\right) \quad (11)$$

Using a linear tire model, the lateral forces are given as

$$F_{yf} = -C_f\alpha_f \quad (12)$$

$$F_{yr} = -C_r\alpha_r \quad (13)$$

where C_f and C_r are the front and rear cornering stiffnesses, respectively. Substituting the expressions for the lateral forces into Equations 1 through 3 and making a small angle approximation for the steering and slip angles yields

$$m\dot{U}_x = mrU_y + F_{xr} + F_{xf} + C_f(\frac{U_y + ra}{U_x})\delta \quad (14)$$

$$m\dot{U}_y = -C_r(\frac{U_y - rb}{U_x}) - C_f(\frac{U_y + ra}{U_x}) - mrU_x + C_f\delta + F_{xf}\delta \quad (15)$$

$$I_z\dot{r} = aF_{xf}\delta - aC_f(\frac{U_y + ra}{U_x}) + bC_r(\frac{U_y - rb}{U_x}) + aC_f\delta + \frac{d}{2}(\Delta F_{xr} + \Delta F_{xf}) \quad (16)$$

Assuming a vehicle that has throttle, brake, and steer-by-wire capabilities so that steering, braking and two drive wheels can be controlled, the equations can be rewritten as

$$M\ddot{q} = f(\dot{q}) + g(\dot{q}, u_c) \quad (17)$$

where $\dot{q} = [U_x\ U_y\ r]^T$ and the control vector $u_c = [\delta\ F_{xrf}\ F_{xlf}\ F_{xrr}\ F_{xlr}]^T$. M is the positive definite mass matrix, $f(\dot{q})$ contains the terms that are not influenced by the control vector and $g(\dot{q}, u_c)$ has the remaining controlled terms.

$$M = \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & I_z \end{bmatrix} \quad (18)$$

$$f(\dot{q}) = \begin{bmatrix} mrU_y \\ -C_r(\frac{U_y-rb}{U_x}) - C_f(\frac{U_y+ra}{U_x}) - mrU_x \\ -aC_f(\frac{U_y+ra}{U_x}) + bC_r(\frac{U_y-rb}{U_x}) \end{bmatrix} \quad (19)$$

$$g(\dot{q}, u_c) = \begin{bmatrix} F_{xr} + F_{xf} + C_f(\frac{U_y+ra}{U_x})\delta \\ C_f\delta + F_{xf}\delta \\ aF_{xf}\delta + aC_f\delta + \frac{d}{2}(\Delta F_{xr} + \Delta F_{xf}) \end{bmatrix} \quad (20)$$

The controlled terms, $g(\dot{q}, u_c)$, are determined by the desired control forces as described in the following section. Once the values of the controlled terms are known, it is possible to solve for the control vector, u_c (Gerdes and Rossetter 2001).

3 Control Law

For this paper the environment is modeled as a straight section of roadway with global coordinates $w = [s\ e\ \psi]^T$ where s is the distance along the roadway, e is the distance of the vehicle's center of gravity from the lane center and ψ is the heading angle (Figure 2). Transformation between the global and body fixed velocities (or coordinates) is achieved with

$$\frac{\partial\dot{w}}{\partial\dot{q}} = \frac{\partial w}{\partial q} = \begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (21)$$

The control law introduced by Gerdes and Rossetter applies a control force to the vehicle using steering and differential braking. This force consists of a conservative portion derived from a potential function and a generalized damping term. For lane-keeping, a simple quadratic potential function is used with the minimum at the lane center. Since the hazards are fixed in the environment (in this case, the lane edges) it makes sense to define the potential relative to environmental coordinates. As mentioned in the introduction, it is necessary to incorporate a projection (lookahead) into the potential function for high speed stability. As a result, the quadratic potential used to generate the desired control force is a function of this projected offset from the lane center, e_{la} (Figure 3).

$$V(e_{la}) = k(e_{la})^2 = k(e_{cf} + x_{la}\sin\psi)^2 \qquad \text{where,} \quad (22)$$

$$e_{cf} = e + x_{cf}\sin(\psi) \quad (23)$$

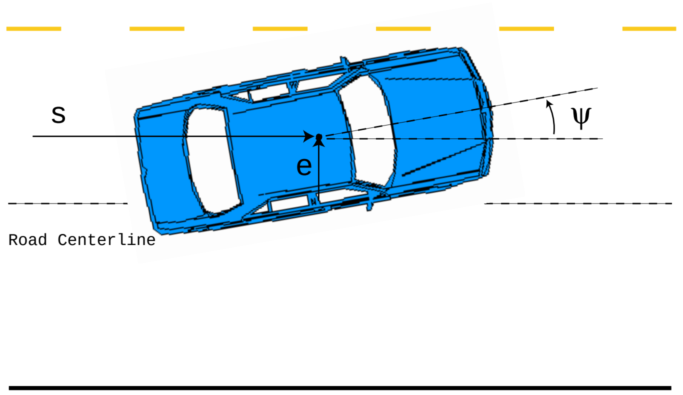

Figure 2. Global Coordinates

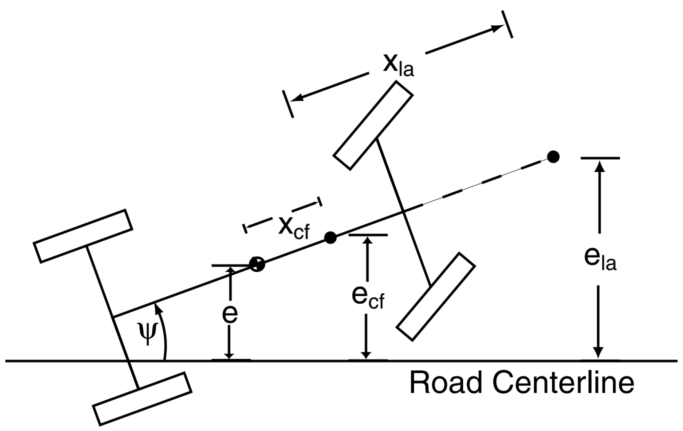

Figure 3. Important Locations

Previous results from linear analysis also show that for stability the control force must be applied in front of the neutral steer point (Rossetter and Gerdes 2000).

$$x_{cf} > \frac{aC_f - bC_r}{C_f + C_r} \tag{24}$$

where x_{cf} is the location of the force (Figure 3). The neutral steer point is the location on a vehicle where an external force creates no steady state yaw rate. Physically, manipulation of the control force location is done by altering the coupling between the lateral and yaw modes through differential braking or four wheel steering.

Allowing the control force derived from the potential to shift from the c.g. adds extra yaw components in the equations of motion. Assuming a potential, V, and a damping term $F(w, \dot{q})$ the control law is

$$g(\dot{q}, u_c) = F(w, \dot{q}) - \left(\frac{\partial V}{\partial w} R \frac{\partial w}{\partial q}\right)^T \tag{25}$$

where

$$R = \begin{bmatrix} 1 & 0 & -x_{cf}\sin\psi \\ 0 & 1 & x_{cf}\cos\psi \\ 0 & 0 & 1 \end{bmatrix} \tag{26}$$

The above control law has transformed the control forces to body fixed coordinates. The equations of motion derived in the previous section can now be written as

$$M\ddot{q} = f(\dot{q}) + F(w, \dot{q}) - \left(\frac{\partial V}{\partial w} R \frac{\partial w}{\partial q}\right)^T \tag{27}$$

In essence, this control law adds conservative forces and damping to the existing vehicle dynamics. Therefore, it is straightforward to show that the sum of the total kinetic energy and the artificial potential energy of the controller are decreasing (Gerdes and Rossetter 2001). For the lanekeeping controller, this bound on total energy is extremely conservative as a bound for the vehicle's lateral motion. This conservatism results from including the large amount of energy in the vehicle's longitudinal direction. Although some of this energy may enter into the lateral and yaw modes of the vehicle, most of it will not. Since the lateral and yaw motions are coupled, a new bounding function will be found that incorporates the global lateral and yaw directions. Since the bounding function involves global coordinates, the pertinent vehicle dynamics will also be transformed to a global frame of reference.

With only a lateral potential in the control law, the equations of motion in terms of the global coordinates e and ψ are

$$m\ddot{e} = -\frac{\partial V(e_{la})}{\partial e} + \left(F_{yr} + \hat{F}_{yf}\right)\cos\psi \tag{28}$$

$$I_z\ddot{\psi} = -\frac{\partial V(e_{la})}{\partial e} x_{cf}\cos\psi - bF_{yr} + a\hat{F}_{yf} \tag{29}$$

where $\hat{F}_{yf}$ is the component of the lateral front force that is not dependent on δ. Substituting the tire forces with small angle approximations on the slip angle from Equations 12 and 13 yields

$$m\ddot{e} = -\frac{\partial V(e_{la})}{\partial e} + -\frac{U_y}{U_x}(C_f + C_r)\cos\psi + \frac{\dot{\psi}}{U_x}(C_r b - C_f a)\cos\psi \tag{30}$$

$$I_z\ddot{\psi} = -\frac{\partial V(e_{la})}{\partial e}x_{cf}\cos\psi + \frac{U_y}{U_x}(bC_r - aC_f) - \frac{\dot{\psi}}{U_x}(C_r b^2 + C_f a^2) \quad (31)$$

Using the transformation from global to body fixed coordinates

$$\dot{e} = U_y cos\psi + U_x sin\psi \qquad \text{giving} \quad (32)$$
$$U_y = \frac{\dot{e}}{\cos\psi} - \frac{U_x sin\psi}{\cos\psi} \quad (33)$$

Substituting this into the dynamics gives

$$m\ddot{e} = -\frac{\partial V(e_{la})}{\partial e} - \frac{\dot{e}(C_r + C_f)}{U_x} + (C_f + C_r)\sin\psi + \frac{\dot{\psi}(bC_r - aC_f)\cos\psi}{U_x} \quad (34)$$
$$I_z\ddot{\psi} = -\frac{\partial V(e_{la})}{\partial e}x_{cf}\cos\psi + \frac{\dot{e}(bC_r - aC_f)}{U_x cos\psi} + (aC_f - bC_r)\tan\psi - \frac{\dot{\psi}(b^2C_r + a^2C_f)}{U_x} \quad (35)$$

These equations will be used to bound the lateral motion of the vehicle.

4 Lyapunov Function for Bounded Hazard

This section will present a Lyapunov function that bounds the lateral motion of the vehicle. The Lyapunov function is an energy like function involving the system states (in this case e, ψ, and their derivatives). Since the derivative of this function is negative semi-definite (or negative-definite) within a region of attraction, the function value within this region will remain constant or decrease. As a result, it is possible to bound the individual states of the function. The Lyapunov function consists of the artificial potential energy, the kinetic energy in the lateral and yaw directions, and terms that are dependent on vehicle heading.

$$L = V(e_{cf}) + \frac{1}{2}m\dot{e}^2 + \frac{1}{2}I_z\dot{\psi}^2 + (bC_r - aC_f)ln(|\sec\psi|) + \frac{1}{2}x_{cf}(C_f + C_r)(\sin\psi)^2 \quad (36)$$

The necessity of the last two terms will be clear when showing the negative semi-definiteness of $\dot{L}$. To be a Lyapunov function, the following two conditions must be satisfied within a local region of the equilibrium.

1. $L(z) > 0$
2. $\dot{L}(z) \leq 0$

In order to satisfy the first condition the last two terms must be positive definite, which puts the following requirement on the location of the control force

$$x_{cf} > \left(\frac{aC_f - bC_r}{C_f + C_r}\right)\left(\frac{ln(|\sec\psi|)}{0.5(\sin\psi)^2}\right) \quad (37)$$

This is an important condition on the location of the control force and deserves some explanation. First, it is interesting to note that

$$\lim_{\psi\to 0}\left(\frac{ln(|\sec\psi|)}{0.5(\sin\psi)^2}\right) = \lim_{\psi\to 0}\left(\frac{\tan\psi}{\sin\psi\cos\psi}\right) = 1 \quad (38)$$
$$\lim_{\psi\to\pm\frac{\pi}{2}}\left(\frac{ln(|\sec\psi|)}{0.5(\sin\psi)^2}\right) = \infty \quad (39)$$

This means that with ψ close to zero, the condition is simply that the control force must be in front of the neutral steer point (the same condition necessary for dynamic stability in a linear analysis). If the heading angle is at $\pm\frac{\pi}{2}$ the vehicle is perpendicular to the gradient of the potential function. Thus, there is no control force location that will influence the vehicle's rotation. Since the terms involving ψ are always positive and equal to or greater than one, the minimum control force location becomes more positive with increasing heading for an oversteering vehicle (because $aC_f > bC_r$) and more negative for an understeering vehicle (where $aC_f < bC_r$). This means that for an understeering vehicle, the condition will always be satisfied if the control force is in front of the neutral steer point. For vehicles with only steering control, this condition is always satisfied. For an oversteering vehicle, a safe location for the application point will depend on the maximum heading the vehicle will achieve.

Assuming that the control force is at an appropriate location, the first condition is satisfied for the Lyapunov function. Now the goal is to show that that the derivative is negative semi-definite.

$$\dot{L} = \frac{\partial V(e_{cf})}{\partial e_{cf}}\dot{e}_{cf} + \dot{e}(m\ddot{e}) + \dot{\psi}(I_z\ddot{\psi}) + (bC_r - aC_f)\tan(\psi)\dot{\psi} + x_{cf}(C_f + C_r)\sin(\psi)\cos(\psi)\dot{\psi} \quad (40)$$

Substituting in the equations of motion from the previous section and grouping the velocity terms yields,

$$\dot{L} = \dot{x}^T Q\dot{x} + \frac{\partial V(e_{cf})}{\partial e_{cf}}\dot{e}_{cf} - \frac{\partial V(e_{la})}{\partial e}x_{cf}\cos\psi\dot{\psi} - \frac{\partial V(e_{la})}{\partial e}\dot{e} + (aC_f - bC_r)\tan(\psi)\dot{\psi} + (C_f + C_r)\sin(\psi)\dot{e} + (bC_r - aC_f)\tan(\psi)\dot{\psi} + x_{cf}(C_f + C_r)\sin(\psi)\cos(\psi)\dot{\psi} \quad (41)$$

where,

$$\dot{x} = \begin{bmatrix} \dot{e} \ \dot{\psi} \end{bmatrix}^T$$

$$Q = \begin{bmatrix} -\frac{(C_f+C_r)}{U_x} & \frac{0.5[(bC_r-aC_f)\cos^2\psi+(bC_r-aC_f)]}{U_x cos(\psi)} \\ \frac{0.5[(bC_r-aC_f)\cos^2\psi+(bC_r-aC_f)]}{U_x cos(\psi)} & -\frac{(b^2C_r+a^2C_f)}{U_x} \end{bmatrix}$$

Assuming a quadratic potential of the form described in Equation 22

$$\begin{aligned} \dot{L} &= \dot{x}^T Q\dot{x} + 2ke_{cf}(\dot{e} + x_{cf}\cos\psi\dot{\psi}) \\ &- 2k(e_{cf} + x_{la}\sin\psi)x_{cf}\cos\psi\dot{\psi} \\ &- 2k(e_{cf} + x_{la}\sin\psi)\dot{e} + (aC_f - bC_r)\tan\psi\dot{\psi} \\ &+ (C_f + C_r)\sin\psi\dot{e} + (bC_r - aC_f)\tan\psi\dot{\psi} \\ &+ x_{cf}(C_f + C_r)\sin\psi\cos\psi\dot{\psi} \end{aligned} \quad (42)$$

Cancelling terms gives

$$\begin{aligned} \dot{L} &= \dot{x}^T Q\dot{x} - 2kx_{la}x_{cf}\sin\psi\cos\psi\dot{\psi} - 2kx_{la}\sin\psi\dot{e} \\ &+ (C_f + C_r)\sin\psi\dot{e} + x_{cf}(C_f + C_r)\sin\psi\cos\psi\dot{\psi} \end{aligned} \quad (43)$$

Notice that if the lookahead distance is chosen to be

$$x_{la} = \frac{(C_f + C_r)}{2k} \quad (44)$$

the derivative of the Lyapunov function is simply

$$\dot{L} = \dot{x}^T Q\dot{x} \quad (45)$$

The final task remaining is to prove that Q is negative definite. This can be done using Sylvester's Criteria to show that -Q is positive definite. The first term is positive by inspection an the determinant of the matrix is

$$\begin{aligned} det(-Q) &= \frac{(C_f + C_r)(b^2C_r + a^2C_f)\cos^2\psi}{U_x^2\cos^2\psi} \\ &- \frac{0.25(bC_r - aC_f)^2(\cos^2\psi + 1)^2}{U_x^2\cos^2\psi} \end{aligned} \quad (46)$$

The denominator is always positive so the goal is to look at the conditions for which the numerator is positive. Obviously, the numerator is not positive for all heading angles, but in fact the condition on the heading angle is quite large for normal vehicle parameters. In order for the numerator to be positive

$$\frac{4(C_f + C_r)(b^2C_r + a^2C_f)\cos^2\psi}{(bC_r - aC_f)^2} > (\cos^2\psi + 1)^2 \quad (47)$$

Since $0 \leq \cos^2\psi \leq 1$ the right hand side is bounded by

$$1 \leq (\cos^2\psi + 1)^2 \leq 4 \quad (48)$$

A conservative region of attraction is given by a heading angle that satisfies

$$\frac{(C_f+C_r)(b^2C_r+a^2C_f)\cos^2\psi}{(bC_r-aC_f)^2} > 1 \qquad \text{or} \quad (49)$$

$$\cos^2\psi > \frac{(bC_r-aC_f)^2}{(C_f+C_r)(b^2C_r+a^2C_f)} \quad (50)$$

Using the approximate parameters for a Mercedes E-class sedan (Table 1), this condition states that

$$|\psi| < 82.8 \text{ Degrees} \quad (51)$$

which is a huge region of attraction. In fact, if this vehicle was at this heading angle relative to the road, the ends would be outside the lane boundary. To have any chance of achieving this heading angle the initial value of the Lyapunov function would have to be extremely large.

$$\begin{aligned} L(0) &> (bC_r - aC_f)ln(|\sec\psi_{max}|) \\ &+ \frac{1}{2}x_{cf}(C_f + C_r)(\sin\psi_{max})^2 \end{aligned} \quad (52)$$

Using an $x_{cf} = 1$, $\psi_{max} = 82.8$ Degrees, and the parameters in Table 1, the initial function value would have to be larger than $2.48 \times 10^5 J$. If the initial heading angle is small, this initial energy has to come from a combination of an initial offset into the potential, a lateral velocity, or a rotational velocity. In order to have the initial energy necessary to achieve ψ_{max} the initial velocities and/or initial incursion into the potential would have to be inordinately large. Therefore, for any practical set of initial conditions, the system remains within the region of attraction and Q is negative definite. Assuming the conditions are met for the lookahead distance and the location of the control force application point, the system hazard is bounded by $L(0)$.

$$L(t) \leq L(0) \quad (53)$$

$$\begin{aligned} L(0) &= V(e_{cf}(0)) + \frac{1}{2}m\dot{e}(0)^2 \\ &+ \frac{1}{2}I_z\dot{\psi}(0)^2 + (bC_r - aC_f)ln(|\sec\psi(0)|) \\ &+ \frac{1}{2}x_{cf}(C_f + C_r)(\sin\psi(0))^2 \end{aligned} \quad (54)$$

For all practical cases a gain can be chosen for the potential that will guarantee avoidance of a lateral hazard. Since the energy in

m (kg)	1860
I_z (N/m^2)	3100
C_f (N/rad)	130000
C_r (N/rad)	160000
a (m)	1.37
b (m)	1.43

Table 1. Vehicle Parameters

$L(0)$ can't increase, the worst case scenario is that it all transfers into lateral hazard. If there is a known lateral obstacle at a position, x_{obst}, then at this position all the energy should be stored in the potential.

$$L(0) = V(e_{cf}) = \frac{1}{2}kx_{obst}^2 \tag{55}$$

Knowing the initial energy $L(0)$ and the position of the obstacle, the potential field gain k can be found to avoid the hazard.

5 Simulation

The simulation uses the three degree of freedom bicycle model with parameters given in Table 1. The vehicle is simulated starting at the lane center, $e(0) = 0m$, with an initial longitudinal velocity of $40m/s$. The initial heading is varied from 1 to 5 degrees and the potential gain is chosen so that the vehicle will never exceed a lateral offset of $0.75m$ (representing the side of the vehicle crossing a lane edge). The potential function used in the simulation is shown in Figure 4 where the minimum is at the lane center. The control force location, x_{cf}, is $1m$ in front of the neutral steer point and the lookahead into the potential is

$$x_{la} = \frac{C_f + C_r}{2k} = 6.6m \tag{56}$$

Figure 5 shows the Lyapunov function in time and as expected it is always decreasing. Of course, as the initial heading is increased the initial energy (or hazard) is also larger. Figure 6 shows the lateral position of the vehicle on the roadway. The gain was chosen so that in the worst case ($\psi(0) = 5$ Degrees) a lateral obstacle at $0.75m$ will be avoided. The results show that this lateral position is never achieved and that the gain based on the initial energy of the Lyapunov function is only conservative by about a factor of two in this case. This slight conservatism can be viewed as a safety factor in the lanekeeping system.

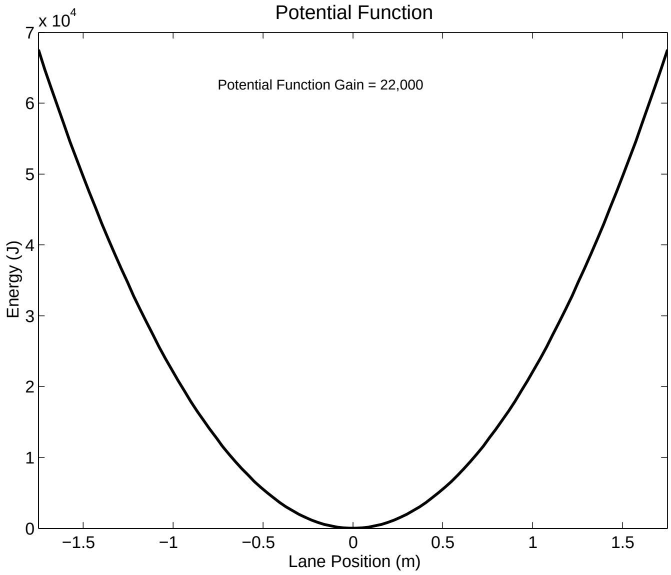

Figure 4. Potential Function

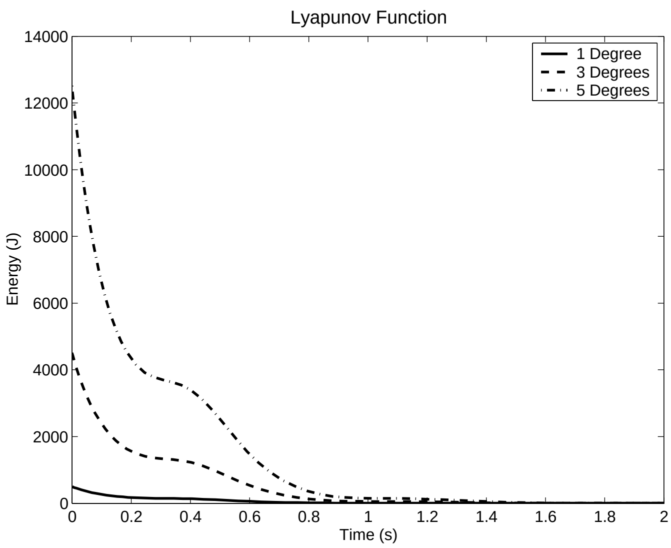

Figure 5. Lyapunov Function

6 Conclusions

This paper presented a Lyapunov function bounding the lateral motion of a vehicle. In order to achieve this bound, two conditions have to be met for the lookahead distance and the location of the control force application point.

1. $x_{la} = \frac{(C_f + C_r)}{2k}$
2. $x_{cf} > \left(\frac{aC_f - bC_r}{C_f + C_r}\right)\left(\frac{ln(|\sec\psi|)}{0.5(\sin\psi)^2}\right)$

As long as these conditions are met, the Lyapunov function gives a worst case intrusion into the artificial potential function. This

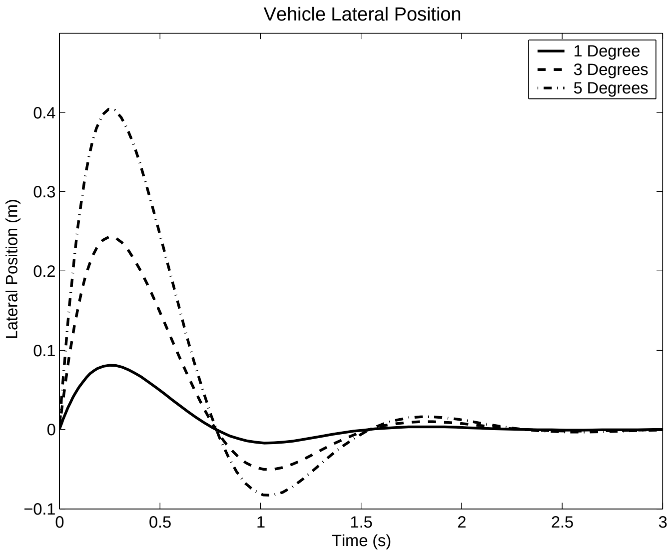

Figure 6. Vehicle Lateral Position

is extremely useful because the potential field can be scaled to guarantee collision avoidance with fixed lateral obstacles. Future work will incorporate road curvature and external disturbances (such as physical forces like sidewind or driver inputs) into the hazard bound. The bounding results presented are currently being experimentally verified on a steer-by-wire test vehicle.

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