An Energy Based Performance Bound for Lanekeeping Assistance with Force Feedback

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This paper uses an energy function to bound the motion of a system consisting of steering based lanekeeping assistance combined with force feedback. By scaling the inertia of the handwheel the system can be represented as mechanical, meaning it can be derived from kinetic and potential energy. The restrictions that result are interpreted in terms of the mechanical force feedback sources present in a conventional vehicle. Experimental implementation shows that this approach can design force feedback schemes with mathematical guarantees of lanekeeping safety.

Topics / Intelligent Driver Assistance: Lane Departure Prevention, Driver Assist Systems, Steering Assistance and Control

1. INTRODUCTION

Lanekeeping assistance holds the promise to save thousands of lives each year by preventing unintended lane departure. According to the U.S. National Highway Traffic Safety Administration, 32 % of vehicle fatalities in 2002 were the result of "failure to keep in proper lane or running off road" [3]. This accounts for about 19,000 deaths each year that could be saved by simply maintaining lane position in the absence of adequate driver steering commands. One promising way to achieve this goal is by using steer-by-wire to actively steer the vehicle towards lane center. Because steer-by-wire has no inherent force feedback on the handwheel, this assistance system must include artificial force feedback to work smoothly with the driver. This force feedback system couples with the vehicle and lanekeeping system, creating a combined dynamic system. As a safety system, we want to guarantee that the vehicle stays in the lane, and thus this entire system performance must be guaranteed with the force feedback active.

Past work [6] identified the need for careful choice of force feedback gains to ensure stability of the system. This work showed that a stable system can be created by recreating the mechanical force feedback sources present in a conventional vehicle. Although stability is necessary, it is also critical to bound the motion of the vehicle to ensure it stays in the lane. In [4] the system is represented as mechanical, meaning it is derived from a potential and kinetic energy and some drift terms. This is possible because the lanekeeping controller is also energy-based, and thus with a few restrictions on system gains the entire system can be derived from energy. With this representation, the road curvature can be viewed as a disturbance affecting the mechanical system. In this way a numerical bound can be found for the motion possible from this disturbance.

This paper applies these energy-based bounding results to a system with force feedback, to bound the combined system of vehicle and handwheel. Even without force feedback, specific requirements must be put on the controller gains to allow the system to be derived from kinetic and potential energy. The introduction of force feedback here creates new difficulties because of the huge difference in the energy of the handwheel compared to that of the vehicle itself. To create the symmetry in the system energy necessary for representation as a mechanical system, the inertia of the handwheel is scaled. This is similar to the more general approach in [1], in which a linear transformation symmetrizes the system. By deriving the dynamics of the system from a potential energy-like function, a Lyapunov function consists of the kinetic plus the pseudo-potential energy. This Lyapunov function bounds the energy of the system, which also provides a bound on the value of each system state. Thus this function can be used to guarantee that the vehicle will stay in the lane, a powerful guarantee for a system of this type.

In order for the dynamics to be derivable from a potential, the force feedback gains must be set

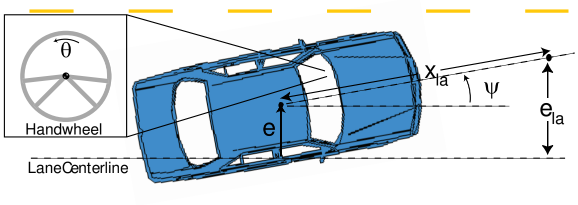

Fig. 1: Lanekeeping System State Definitions

to specific values. These requirements can be interpreted in terms of the mechanical force feedback sources in a normal vehicle (aligning moment, jacking torque, etc) to provide design intuition. With these requirements on the gains met, this Lyapunov function guarantees stability of the system, but more importantly it can be used to bound the motion and guarantee lanekeeping performance with the force feedback system active. Utilizing the bounding approach of [4], the curvature of the road is viewed as a disturbance to the system, and a bound is found using the characteristics of the road itself. Experimental results confirm the utility of this approach.

2. VEHICLE DYNAMICS WITH LANEKEEPING AND FORCE FEEDBACK

The lanekeeping system considered uses steer-by-wire to adjust the steering angle of the vehicle to push the vehicle back toward lane center [5]. As shown in Figure 1 the controller projects the heading error ψ of the vehicle forward by a distance x_{la}. The steering angle added to the driver command by the lanekeeping assistance to push the car back to lane center is based on the product of the projected error with a gain k_p:

$$\delta_p = \frac{k_p}{C_f}(e + x_{la}\psi) \tag{1}$$

To work smoothly with the driver this system requires force feedback on the handwheel. The force feedback consists of a torque on the handwheel proportional to a linear combination of the vehicle and handwheel states. The goal is to bound the underlying system, thus we consider the dynamics without any user input. This system is specifically intended to help maintain lane position when the driver is unable to steer, so we must guarantee operation in this situation.

Because the lanekeeping system aims to assist the driver rather than take over control, the handwheel is still active as an input device to steer the roadwheels. As such, any movement of the handwheel caused by the force feedback steers the roadwheels, and any change in the vehicle states affects the force feedback on the handwheel. Thus the vehicle model must include both the vehicle with lanekeeping and the handwheel with force feedback. The longitudinal speed is not included, because this Lyapunov function does not include the longitudinal energy. This restriction to lateral and heading dynamics has been shown to reduce the conservatism associated with a total energy function for the vehicle [4].

The vehicle model uses six states: four representing the position and heading of the vehicle and its derivatives, and two for the handwheel position and rate. With these states, the bicycle model can be combined with a second order system representing the force feedback. Together these form a model of the combined system of lanekeeping and force feedback. The dynamics for this system are:

$$\begin{bmatrix} \dot{x} \\ \ddot{x} \end{bmatrix} = A \begin{bmatrix} x \\ \dot{x} \end{bmatrix} \qquad x = \begin{bmatrix} e & \psi & \theta \end{bmatrix}^T \tag{2}$$

where the A matrix is given by

$$\begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ -\frac{2k_p}{m} & \frac{(C_f+C_r)-2kx_{la}}{m} & \frac{C_f}{ms_r} & \frac{-(C_f+C_r)}{mU_x} & \frac{(-aC_f+bC_r)}{mU_x} & 0 \\ \frac{-2k_p a}{I_z} & \frac{(aC_f-bC_r)-2kx_{la}a}{I_z} & \frac{aC_f}{ms_r} & \frac{(-aC_f+bC_r)}{I_zU_x} & \frac{-(a^2C_f+b^2C_r)}{I_zU_x} & 0 \\ \frac{k_e}{I_w} & \frac{k_\psi}{I_w} & \frac{k_\theta}{I_w} & \frac{k_{\dot{e}}}{I_w} & \frac{k_{\dot{\psi}}}{I_w} & \frac{k_{\dot{\theta}}}{I_w} \end{bmatrix}$$

Here C_f and C_r are the cornering stiffnesses for the front and rear axles, a and b are the distances from the CG to the front and rear axles respectively, U_x is the vehicle's longitudinal speed, and s_r is the steering ratio. m, I_z, and I_w are the mass of the car and moments of inertia of the car and the handwheel. The k_i in the last row are individual gains for the general state feedback that produces the force feedback. These gains will be determined in the following section.

The goal of this work is to find a Lyapunov function for this system to ensure stability and obtain numerical bounds on the motion. One way to do this is to define the system in terms of energy by writing it as a Lagrangian system. In general form such a system can be written:

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{q}} + \frac{\partial V}{\partial q} \quad = \quad F \tag{3}$$

Here q_i are the generalized coordinates, T is the kinetic energy, and V is the potential energy. For the lanekeeping system, this results in the following:

$$\begin{bmatrix} m & 0 & 0 \\ 0 & I_z & 0 \\ 0 & 0 & G_sI_w \end{bmatrix} \begin{bmatrix} \ddot{e} \\ \ddot{\psi} \\ \ddot{\theta} \end{bmatrix} \quad = \quad f_d(\dot{x}) - \begin{bmatrix} \frac{\partial \tilde{V}}{\partial e} \\ \frac{\partial \tilde{V}}{\partial \psi} \\ \frac{\partial \tilde{V}}{\partial \theta} \end{bmatrix} \tag{4}$$

This expression says that the dynamics are a function of a potential energy $\tilde{V}$, and a drift term f_d. The drift term f_d is purely a function of the velocity states. The potential energy forces a symmetry to the dynamics, as can be seen by writing out the derivative from Equation 4 with a quadratic potential function.

$$\tilde{V} = c_1 e^2 + c_2 e\psi + c_3\psi^2 + c_4\theta^2 + c_5\theta e + c_6\theta\psi \quad (5)$$

$$\begin{bmatrix} \frac{\partial \tilde{V}}{\partial e} \\ \frac{\partial \tilde{V}}{\partial \psi} \\ \frac{\partial \tilde{V}}{\partial \theta} \end{bmatrix} = \begin{bmatrix} 2c_1 & c_2 & c_5 \\ c_2 & 2c3 & c_6 \\ c_5 & c_6 & 2c_4 \end{bmatrix} \begin{bmatrix} e \\ \psi \\ \theta \end{bmatrix} \quad (6)$$

Thus regardless of the exact values of the potential coefficients, the system dynamics must have a symmetry to be derived from a potential. To handle the large magnitude difference between the vehicle energy and the handwheel energy, the handwheel inertia is scaled by factor G_s. This is similar to other approaches, such as [1, 2] in which specific linear algebraic requirements are placed on system matrices to allow for symmetrizability. Here the requirements are simply made explicit and specific to the form of the system.

With the potential $\tilde{V}$ always positive and the drift terms always dissipative stability is guaranteed. With these requirements, a Lyapunov function for the system is given by:

$$L = \tilde{V} + \frac{1}{2}\dot{x}\tilde{M}\dot{x} \quad (7)$$

Clearly this function is always positive with $\tilde{V}$ positive, as the mass matrix is always positive definite. The function can be shown to be always decreasing by taking the derivative and inserting the dynamics from Equation 4.

$$\begin{aligned} \frac{dL}{dt} &= \dot{x}\begin{bmatrix} \frac{\partial \tilde{V}}{\partial e} \\ \frac{\partial \tilde{V}}{\partial \psi} \\ \frac{\partial \tilde{V}}{\partial \theta} \end{bmatrix} + \dot{x}\tilde{M}\ddot{x} && (8) \\ &= \dot{x}\begin{bmatrix} \frac{\partial \tilde{V}}{\partial e} \\ \frac{\partial \tilde{V}}{\partial \psi} \\ \frac{\partial \tilde{V}}{\partial \theta} \end{bmatrix} + \dot{x}\left(f_d(\dot{x}) - \begin{bmatrix} \frac{\partial \tilde{V}}{\partial e} \\ \frac{\partial \tilde{V}}{\partial \psi} \\ \frac{\partial \tilde{V}}{\partial \theta} \end{bmatrix}\right) && (9) \\ &= \dot{x} f_d(\dot{x}) && (10) \end{aligned}$$

Thus with the drift term dissipative the Lyapunov function is decreasing for all system states as desired.

3. REQUIREMENTS ON THE GAINS

3.1 The Potential

The potential must be a function of the positional states e, ψ, θ. In general form this potential is:

$$\tilde{V} = c_1 e^2 + c_2 e\psi + c_3\psi^2 + c_4\theta^2 + c_5\theta e + c_6\theta\psi \quad (11)$$

As a potential, these function coefficients must be chosen such that the function is positive for all system states. For use as part of the Lyapunov function, the potential must also reflect the dynamics, as in equation 4. This means the derivative of this potential must match the position dependent terms in the dynamics. Matching coefficients and enforcing the positive definite nature of the potential, the following conditions must be met:

$$\begin{aligned} c_1 &= k_p && (12) \\ c_2 &= 2k_p a && (13) \\ c_3 &= k_p a(x_{la} + a) + \frac{1}{2}(aC_f - bC_r) && (14) \\ c_4 &= -\frac{k_\theta}{2} \geq \frac{{C_f}^2}{4{s_r}^2 k_p} = (n+1)\frac{{C_f}^2}{4{s_r}^2 k_p} && (15) \\ c_5 &= -k_e = -\frac{C_f}{sr} && (16) \\ c_6 &= -k_\psi = -\frac{aC_f}{sr} && (17) \end{aligned}$$

Here n is a parameter greater than 0 which represents one of the degrees of freedom in the force feedback system. With these six requirements the dynamics can be derived from a positive potential.

3.2 The Drift Term

The other requirement for the Lyapunov function is that the drift term f_d is dissipative, meaning it is dissipating energy for all nonzero system states. The drift term is simply all of the velocity dependent forces, effectively all of the forces in columns 2, 4 and 6 of the A matrix in equation 2. Mathematically this means:

$$-\begin{bmatrix} \dot{e} & \dot{\psi} & \dot{\theta} \end{bmatrix} f_d \geq 0 \quad (18)$$

$$\begin{bmatrix} \dot{e} \\ \dot{\psi} \\ \dot{\theta} \end{bmatrix}^T \begin{bmatrix} \frac{C_f+C_r}{U_x} & \frac{aC_f-bC_r}{U_x} & -\frac{1}{2}k_{\dot{e}} \\ \frac{aC_f-bC_r}{U_x} & \frac{a^2C_f+b^2C_r}{U_x} & -\frac{1}{2}k_{\dot{\psi}} \\ -\frac{1}{2}k_{\dot{e}} & -\frac{1}{2}k_{\dot{\psi}} & -k_{\dot{\theta}} \end{bmatrix} \begin{bmatrix} \dot{e} \\ \dot{\psi} \\ \dot{\theta} \end{bmatrix} \geq 0 \quad (19)$$

This matrix is positive definite if all submatrices have positive determinants. The first entry is positive, and first submatrix has positive determinant as shown by previous work. The determinant for the entire matrix is positive if the following condition is met:

$$-k_{\dot{\theta}}\frac{C_f C_r L^2}{U_x} - k_{\dot{\psi}}^2\frac{C_f + C_r}{4} + k_{\dot{\psi}}(aC_f - bC_r) - k_{\dot{e}} k_{\dot{e}}^2 \frac{b^2C_r + a^2C_f}{4} \geq 0 \quad (20)$$

4. THE LYAPUNOV FUNCTION

With the requirements of the previous sections satisfied, a Lyapunov function consists of the potential plus kinetic energies, which are each positive (Eqn. 7). Because the drift terms are dissipative, this function will be decreasing for all system trajectories when the gains are set by the criteria developed in the previous section. Figure 2 shows the system trajectory and Lyapunov function values for a system that exactly meets the requirements of the

Parameter (units)	Value
U_x (m/s)	15
P.F. gain k (N/m)	5000
x_{la} (m)	32
m (kg)	1540
I_z (kgm^2)	2500
C_f (N/rad)	160000
C_r (N/rad)	160000
a (m)	1.3
b (m)	1.3
I_w (kgm^2)	.084
s_r	16
k_θ Nm/rad)	1.49
k_e (N/rad)	1.0
k_ψ (N/rad)	1.33
$k_{\dot\theta}$	0.0001
$k_{\dot e}$	0
$k_{\dot\psi}$	0

Table 1: Controller And Vehicle Parameters

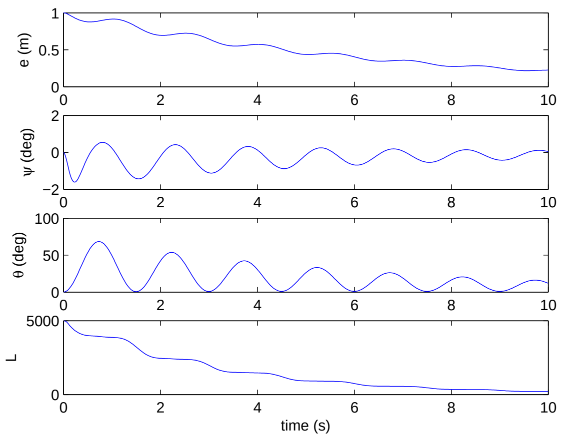

Fig. 2: System Trajectory and Lyapunov Function Values With Minimum Stability Requirements Met (Table 1)

previous two sections. The parameters for this vehicle are given in Table 1. With an initial offset from lane center of 1m, the vehicle does return to lane center, but in a slow yet very oscillatory way. The Lyapunov function does decrease, but clearly this response is unacceptable. Thus we seek a more intuitive way to choose these gains.

5. TRANSLATING GAIN REQUIREMENTS

The gain requirements on each state in the previous section assure stability, but say nothing about how the system will feel to the user. To address this question it is useful to translate these requirements into the force feedback sources present in a conventional vehicle. Here we consider the dominant three sources present in a conventional vehicle: aligning moment, jacking torque and damping [6]. We also consider force feedback based on the amount of lanekeeping assistance being given.

In terms of the states of the system these four torques are:

$$\tau_{align} = k_{align}\left(\frac{2k_p}{C_f}e + \frac{1}{U_x}\dot{e} + (\frac{2k_p x_{la}}{C_f} - 1)\psi + \frac{a}{U_x}\dot\psi + \frac{-1}{s_r}\theta\right) \quad (21)$$

$$\tau_{damp} = k_{damp}\left(-\dot\theta\right) \quad (22)$$

$$\tau_{pf} = k_{pf}\left(-2k_p e - 2k_p x_{la}\psi\right) \quad (23)$$

$$\tau_{jack} = k_{jack}\left(\frac{2k_p}{C_f}e + \frac{2k_p x_{la}}{C_f}\psi - \frac{1}{s_r}\theta\right) \quad (24)$$

The requirements of Equations 12-17 and 20 can be translated into requirements on the force feedback gains k_{align}, k_{damp}, k_{pf} and k_{jack}:

$$k_{align} = \frac{C_f}{2k_p s_r}\frac{(C_f + C_r)}{G_s} \quad (25)$$

$$k_{pf} = \frac{C_f}{2k_p s_r}\frac{n}{G_s} \quad (26)$$

$$k_{jack} = \frac{C_f}{2k_p s_r}\frac{(nC_f - C_r)}{G_s} \quad (27)$$

$$k_{damp} \geq \frac{C_f}{2k_p s_r}\frac{(C_f + C_r)^2}{8k_p s_r U_x G_s} \quad (28)$$

Here $n \geq 0$ is the factor from Equation 12 that can be chosen.

Thus there are three degrees of freedom in the force feedback system, even with the constraint that the dynamics are derived from kinetic and potential energy. First, the factor n can be chosen to adjust the amount of the different types of force feedback. Larger n gives more jacking effect and lanekeeping assistance feedback. Second, the amount of handwheel damping can be chosen to be anything larger than the minimum value in Equation 28. The other degree of freedom is the scaling of the entire force feedback, G_s.

Figure 3 shows a system trajectory with the force feedback parameters from Table 2. Despite the restrictions imposed to make the system mechanical these parameters result in reasonable gains on the mechanical force feedback sources. These are similar to the values for the stock Corvette, with the exception of the damping, which is about twice the stock value. This could be remedied by increasing the inertia scaling G_s, but this additional damping may not be a problem at all, as for highway situations it is often desirable to add some damping to an underdamped vehicle. With these parameters the system returns to lane center smoothly and without large handwheel motion.

6. BOUNDING THE SYSTEM

This analysis constructs a Lyapunov function for the system with proper choice of force feedback

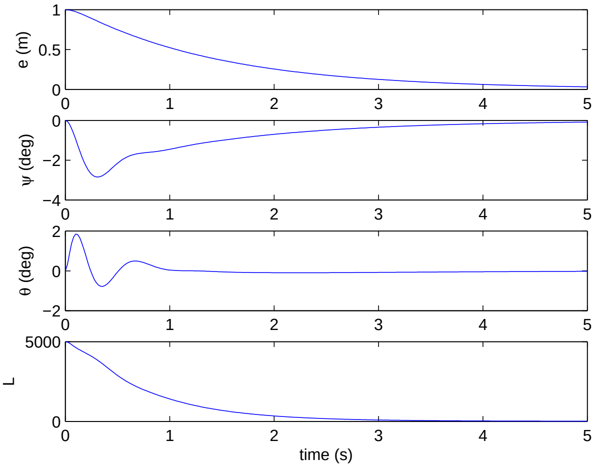

Fig. 3: System Trajectory and Lyapunov Function Values With Mechanical Force Feedback Gains

Parameter (units)	Value
G_s	2000
n	2
k_{damp} (Nm/rad/s)	2.84
k_{jack} (Nm/deg)	0.49
k_{align} (Nm/deg)	1.14
k_{pf} (Nm/m)	3.11

Table 2: Force Feedback Parameters

parameters, guaranteeing stability. This function can also be used to numerically bound the motion of the car. Thus for a specific choice of lanekeeping controller gain, a guarantee can be provided that the vehicle stays in the lane.

This bounding follows the process outlined in [4], the details of which are beyond the scope of this paper. First, for a given disturbance, a change of variables is performed to the equilibrium configuration of the system. Any changes in the disturbance are then viewed as excitation to the system relative to the equilibrium configuration. To provide exponential stability to the equilibrium, a small "cross term" relating to the product of the velocity and position states is added to the Lyapunov function. Writing the potential as a matrix quadratic form $\tilde{V} = x^T P x$, the Lyapunov function with cross term can be written as:

$$L_\epsilon = \begin{bmatrix} x^T & \dot{x}^T \end{bmatrix} \begin{bmatrix} P & \epsilon\frac{PM}{2} \\ \epsilon\frac{MP}{2} & M \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \end{bmatrix} \quad (29)$$

With this cross term chosen such that the Lyapunov function still meets the requirements, this means that the system trajectory approaches the disturbance-caused equilibrium at an exponential rate in the absence of changing disturbance. The rate of exponential convergence and the conservatism depend on this epsilon value, and it is chosen to minimize conservatism while still resulting in

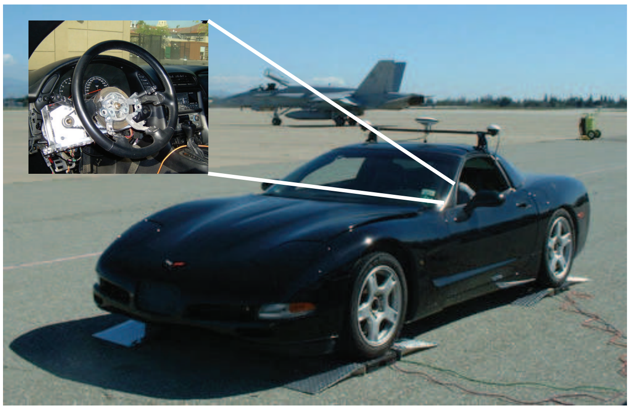

Fig. 4: Experimental Vehicle with Force Feedback System

a valid Lyapunov function. Thus a bound on the system energy can be obtained, and using linear algebra this bound can be converted to a worst-case bound on each of the system states.

7. EXPERIMENTAL RESULTS

This bound has been verified experimentally in the steer by wire vehicle shown in Figure 4. This vehicle is equipped with a high performance force feedback system and lanekeeping assistance based on the Global Positioning System. Details of this experimental implementation of lanekeeping are in [5]. The force feedback system uses a brushless DC motor to provide up to 20Nm of torque on the handwheel. Using a combination of GPS and INS determines the position and heading of the vehicle to about 2cm and .5 degrees with a bandwidth of about 100Hz. This state estimate is compared to a polynomial-based map of the track stored onboard the vehicle, and control is performed at a sampling rate of 200Hz.

To simulate a highway situation, a track with large radius curves was created on an airfield. The track is shown in Figure 5. With the vehicle moving at a constant 33mph (15m/s), the driver removed his hands from the handwheel, and allowed to car to drive with no user input for one complete lap. The curvature of the track, which enters the bounding process as the disturbance, is shown in Figure 6.

Figure 7 shows the response of the system on the track, showing the good match between reality and the simple model used for this analysis. There is some discrepancy, particularly in the handwheel position, likely caused by unmodeled friction in the handwheel system. The numerical bound on e resulting from this set of conditions is 3.15 m, and that on θ is 2.24 rad. Thus in this case the bound is conservative by factors of 1.5 and 1.7 for e and θ respectively. This conservatism is due to the fact that the bound is based on the maximum disturbance on the track (the tightest curve and greatest change in curvature) and this track is not the worst-case track

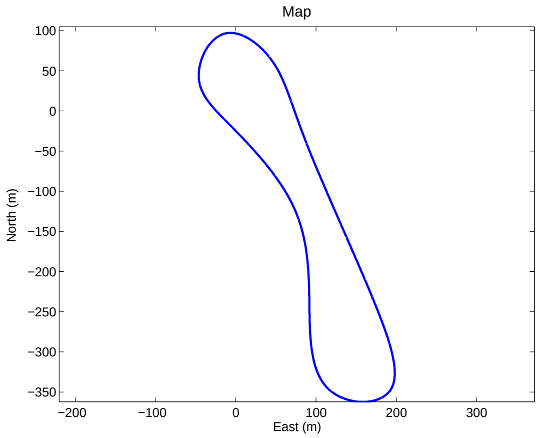

Fig. 5: Lane Track for Experimental Results

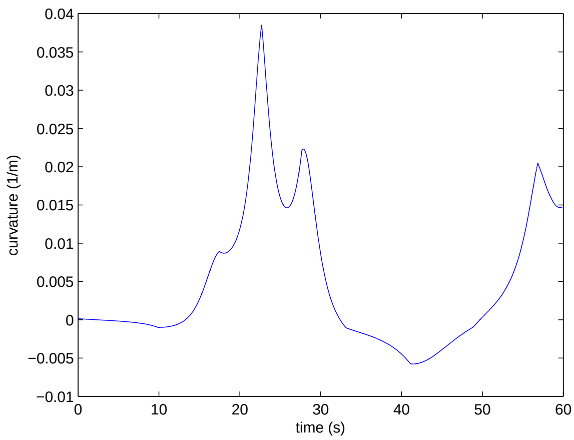

Fig. 6: Curvature Disturbance Input on Experimental Map

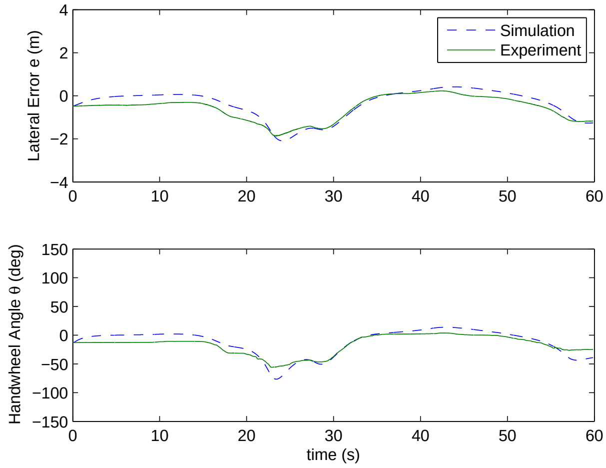

Fig. 7: Experiment and Simulation of Lanekeeping with Force Feedback

with these maximum characteristics. To guarantee that the vehicle stays in the lane for this track, a slightly higher gain would have to be chosen. Despite the small amount of conservatism it is clearly a tight enough bound to be useful for designing the lanekeeping system to match a given road.

8. CONCLUSIONS

The technique described here allows for construction of an analytical Lyapunov function for a vehicle with lanekeeping assistance and force feedback. To achieve this, the dynamics of the system must be symmetrized by scaling the inertia of the handwheel. With proper choice of force feedback parameters the system can then be represented as a mechanical system, and a numerical bound can be calculated on the possible deviation from disturbance such as road curvature. This allows for design of the force feedback while simultaneously guaranteeing that the vehicle will stay in the lane in the absence of driver input.

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